Problem Set 1, String Phenomenology

September 22, 2008

Please hand in solutions to Marcus by Fri, Sep 26. Updates on `www.physto.se/~mberg/minicourse`.

1 Math Warmup: ϑ and E_{2n}

a) We want to find an object $a(\nu, \tau)$ that depends on two complex variables ν and τ and that satisfies the "line bundle relations"

$$a(\nu+1,\tau) = a(\nu,\tau) \tag{1}$$

$$a(\nu+\tau,\tau) = e^{-2\pi i\nu-\pi i\tau}a(\nu,\tau) \tag{2}$$

Make the sum ansatz

$$a(\nu,\tau) = \sum_{n=-\infty}^{\infty} c_n z^n \qquad (z = e^{2\pi i\nu}) \tag{3}$$

and calculate c_n in terms of $q = e^{2\pi i\tau}$. You'll see that a is the Jacobi ϑ_3 function. How do you need to modify the line bundle relations to get ϑ_1, ϑ_2 or ϑ_4?

b) If $E_{2n}(\tau)$ is the normalized Eisenstein series for fixed $n > 1$,

$$E_{2n}(\tau) = 1 + b_n \sum_{k=1}^{\infty} \sigma_{2n-1}(k)q^k \tag{4}$$

where $b_n = (2\pi i)^{2n}/((2n-1)!\,\zeta(2n))$, and $\sigma_{2n}(k)$ is the divisor function

$$\sigma_{2n}(k) = \sum_{d|k} d^{2n} \tag{5}$$

(i.e. d divides k), compute the integral over the positive real line

$$\int_0^\infty d\ell\ \ell^{s-1}(E_{2n}(i\ell)-1)\ . \tag{6}$$

What happens if you take $n \to 1^+$?

2 Extracting effective actions from the string S-matrix

a) Calculate the following chunk of an S-matrix element of two open string vectors:

$$\xi^1_\mu p^1_\nu \xi^2_\sigma p^2_\rho : \psi^\nu(w_1)\overbrace{\psi^\mu(w_1) :: \psi^\sigma(w_2)\psi^\rho}(w_2) : \tag{7}$$

where p^i_μ is the spacetime momentum of external state $i = 1, 2$ and ξ^i_μ is the polarization vector of external state $i = 1, 2$, in terms of the fermion one-loop Green's function $G_F^{\alpha,\beta}$ discussed in class:

$$\overbrace{\psi^\mu(w_1)\psi^\nu}(w_2) = \eta^{\mu\nu}G_F^{\alpha,\beta}(w_1-w_2) \tag{8}$$

and show that the combination of p's and ξ's is reproduced by linearizing the off-diagonal kinetic term for two nonabelian gauge fields A^μ_1 and A^μ_2,

$$\mathcal{L}_{\rm eff} = \frac{1}{2g^2}\eta_{\mu\rho}\eta_{\nu\sigma}{\rm tr}\, F_1^{\mu\nu}F_2^{\rho\sigma}\ . \tag{9}$$

b) Does it matter that the gauge fields are nonabelian?

c) Explain physically why there are no terms like $p_1 \cdot \xi_1$.