Problem Set 2, String Phenomenology

October 10, 2008

Please hand in solutions to Marcus by Tue, Oct 7. Updates on `www.physto.se/~mberg/minicourse`.

1 More math: How to avoid some hard calculations

Convince yourself, and me, that the $\vec{\alpha} = (0,0)$ fermion Green's function satisfies

$$(G_F^{(0,0)}(\nu,\tau))^2 = \mathcal{P}(\nu,\tau) - e_2(\tau) \tag{1}$$

where the semiperiod e_2 is $e_2(\tau) = \mathcal{P}\left(\frac{1+\tau}{2},\tau\right)$. (Warning: this is sometimes called e_3.) In terms of theta functions this is $e_2(\tau) = -4\pi i \partial_\tau \ln(\vartheta_3(0,\tau)/\eta(\tau))$. If you do it "brute force", focus on the first two terms in a ν Laurent expansion. If you do it the "algebraic geometry" way, you'll see it for all ν. (Eq.(1) is true also for $\vec{\alpha} = (0,1/2),(1/2,0)$, with other semiperiods e_i — in fact, this is why it's useful.)

2 "Easy" loop correction: K, partially twisted strings

Here the fermionic part of the integrand doesn't depend on ν. (This isn't immediately manifest, though; one can show it directly like eq. (2.61), in `hep-th/0508043`, or by (1) above as in `hep-th/0204153`.)

Let's consider Kähler potential corrections, i.e. a closed string two-point function. Read section 2.5, up to eq. (2.61), in `hep-th/0508043`, then fill in the steps in the calculation in (2.61)-(2.65), as follows. The open string field A is what I usually call ϕ. The fields A, T and U, are all constant with respect to this worldsheet calculation. Even if there is some notation you don't understand, see if you can perform these key steps of the calculation anyway.

a) semiclassical zero-modes Z_{class} in (2.61) (see 2.49), where $Z = Z_{\text{class}} + Z_{\text{qu}}$.

b) quantum correlator $\langle Z_{\text{qu}} Z_{\text{qu}} \rangle$ in (2.61) using (2.55) and the "torus trick" (2.62).

c) integral $\int_0^\infty d\ell$ in (2.64). Note that this scary-looking integral is actually very simple. Contrast with the analogous integral for the gauge kinetic function f discussed in class.

d) Recalling our $\mathbb{R}^2$ vs. $\mathbb{C}$ torus, show

$$\vec{n} \cdot \vec{a} = \frac{A(n+m\bar{U}) - \bar{A}(n+mU)}{U-\bar{U}} \ . \tag{2}$$

Some phenomenological implications of this Kähler potential correction will be/have been discussed in class.

3 "Hard" loop correction: K, completely twisted strings

see upcoming paper (M.B., Haack, Kang)!