Reading guide to Polchinski: sketches of answers

BEFORE READING: WRITE DOWN SOLUTIONS. GIVE YOURSELF $0-2$ POINTS!

Chapter 1

1. Massive particles are timelike, i.e. their momentum four-vectors point in the time direction. (Polchinski also discusses the generalization to massless particles.) Whether the square of a timelike vector is -1 or $+1$ depends on the convention for the metric, but in any case the square is constant. Now, all functions have a constant value at a given point, but that does not mean the derivative is zero. What the variational principle really means is that δ introduces a parameter (*not* the variable of integration λ, the integral once performed doesn't depend on λ, so we couldn't vary it usefully!), call it ϵ, that lets you vary across a "family" of different trajectories nearby, and only for the trajectory that satisfies the equation of motion is is the derivative really zero, $\partial S/\partial \epsilon = 0$.[1]

   You can imagine that *all* trajectories in this family have their four-momentum normalized to -1, although that is not necessary — you would need to add a constraint using a Lagrange multiplier. If you were curious about test particles falling into a black hole, take a look at Wikipedia: Schwarzschild geodesics, or Kepler problem in GR.

2. Yes there are massless bosons in 2D, but they aren't Goldstone bosons. It is for the reason discussed here, though, that X is not easy to understand, and we tend to use e^X or ∂X in this course, rather than X itself. If G_{XX} is $-\ln r$ (which increases for $r \to \infty$) then $G_{\partial X \partial X}$ is something like $\partial^2(-\ln r) = 1/r^2$, which decreaeses for $r \to \infty$, so is nicer than G_{XX}.

3. (One way to answer the detective question: North Cemetery is across lake Brunnsviken from the Stockholm University physics department!) According to the models where we live on a hyperplane in the extra dimensions, one reason we may not have noticed that gravitons "leak" out into the extra dimensions is that they themselves notice gravity, so if the hyperplane we live on in the extra dimensions has energy/matter on it, it provides a gravitational potential well that keeps things localized closed to our hyperplane. (One simple example of a gravitational well used in these papers is Anti-de-Sitter space, more about which later.) This sounds far-fetched, until you learn how to *calculate* the gravitational source energy of a D-brane, so the hyperplane being gravitationally attractive is more inevitable than it is far-fetched.

   That was gravitons, what about ordinary matter? It would be even more obvious if matter could escape into huge extra dimensions than if gravity could. The explanation is somewhat similar to the above gravitational explanation from the point of view of field theory, but in string theory there is a strikingly different way to view ordinary matter: the D-brane is defined as the place where open strings end, i.e. are stuck (Dirichlet boundary condition), and the endpoints can be our quarks and leptons. So if matter is stuck to the D-brane, that's another reason why we may not have noticed matter particles leaking out into the extra dimensions: they are simply stuck.

   And, there is a connection between the two ideas: if 1. a particle in our universe is a string endpoint and, 2. gravitons are made from closed string loops. Then, endpoints should be able to annihilate with anti-endpoints (!) and form a string without endpoints, i.e. a closed string: a Kaluza-Klein graviton that can escape from our hyperplane. There are indeed phenomenological models of couplings between matter and Kaluza-Klein gravitons, that would allow the production of the latter at particle colliders, though at the moment noone has found any.

4. In the footnote, Polchinski says: we can prove that trajectories do not self-intersect for the point particle, simply by causality (trajectories in spacetime can't bend back and cross themselves), and things are presumably OK for string theory too for the same reason. In other words, *this is one of those fine points not to dwell on*, that can be understood as "personally, I don't worry much".

[1] This language is used, for example in the book by Choquet-Bruhat et al, "Analysis, Manifolds and Physics", or in Wald's GR book.

Contrast with the next question: if you first Wick rotate to avoid Minkowski space, you can't use causality to argue that self-crossing never happens!

5. An oscillating string of zero thickness does develop cusps fairly frequently when flopping around. Zwiebach gives lots of details in his book Chapter 7.5: *Motion of closed strings and cusps*.

   Incidentally, here is a faulty argument why the string oscillation must be smooth: we can expand $X^\mu(\sigma,\tau)$ in Fourier basis waves, and they are smooth. But Fourier series can build up a wave that has discontinuous derivative, like a sawtooth wave. An infinite superposition of smooth functions does not need to be smooth.

   For an ordinary rope, none of this matters, since there is a natural "ultraviolet cutoff": the superposition is a finite sum. This is because waves on a rope that are shorter than the constituents of the rope cannot propagate along the rope. But this does not help with the "fundamental" string we discuss here, that is infinitely thin and has no further constituents. If you worry a little about this, you're in good company, so does e.g. 't Hooft (Nobel prize 1999). I was at a a talk by him in 2004, and after the talk, Polchinski argued that it is quantum physics that "cuts off" such classical singularities. (Unfortunately the question sessions seems to not be in the recording.) I think Polchinski ultimately only meant how the existence of discrete energy packets (photons) lead to Planck's radiation law, that solves the ultraviolet catastrophe of Rayleigh-Jeans law. In string theory, we see this implemented in (at least) two ways: first, an actual corner on the worldsheet would change topology (decrease Euler characteristic, see below), so can be viewed as a quantum effect (see later). Secondly, as we will see in Ch.3, it is also true that Weyl invariance (Ch.2) restricts possible spacetime momenta so they cannot add up to produce a delta function, which prohibits contact interaction on the worldsheet. In all, as far as I am aware, cusp formation does not cause "problems" in (quantum) string theory.

   Perhaps closer to the point is footnote 3 on p.15: we partially worry about the "outer" (extrinsic) geometry as opposed to the "inner" (intrinsic) geometry that appears in the Polyakov action. According to the foonote, invariants from extrinsic geometry cannot appear in a regular action for an infinitely thin string. On the other hand, macroscopic cosmic strings [2] experience exactly this effect, that is manifested as a sharp packet of radiation ("crack of a whip" sound wave, for actual whips) that could give a gravitational wave signal (see video). Polchinski used to illustrate this with an actual whip, to great effect, unfortunately not in this video.

   Incidentally, it is a somewhat unfinished subject in both quantum field theory and string theory how to carefully treat the difference between Lorentzian (Minkowski space) and Riemannian signature[3]. For example, functional integrals in Minkowski space are not considered very well-defined by some matematicians (Ch.1 in my PhD thesis!), and the corresponding Feynman $i\epsilon$ prescription could be made more explicit, as Witten asks for [4].

   Here is a eulogy for Torsten Ekedahl from 2011, that relates mathematicians comparing him to renowned Swedish mathematician Hörmander (what did he do?). I brought up Ekedahl here because like many mathematicians, he was somewhat fascinated by string theory, and I encourage students to interact with mathematicians, and take their comments seriously, because often they have a point.

6. The adjugate matrix is the inverse times the determinant. Actually I often use the adjugate matrix in symbolic computation, for the trivial reason that matrix inversion sometimes gives

[2] Chapter 7.6 in Zwiebach's book

[3] It's often called "Euclidean signature" in physics, even for curved space, but "Euclidean space" in mathematics specifically refers to *flat* space, for which the Euclidean parallel axiom holds, so a safer word for metric signature of all the same sign is "Riemannian" (which is "non-Euclidean" in mathematics — hopefully now you see my point!). Mathematicians refer to what we call "Lorentzian" signature as "pseudo-Riemannian", but I won't use this. You could argue that "Lorentzian" also specifically refers to flat space, but I choose to be less pedantic there: Hendrik Lorentz was a physicist, so the risk for confusion is smaller.

complicated-looking expressions just because the inverse of the determinant has been multiplied into every entry. Keeping the inverse determinant separate amounts to working with the adjugate instead of the inverse, and it is quite possible that the adjugate looks a lot nicer in intermediate steps.

7. Yes, the same for pyramid and sphere as for cube: $\chi = 2$. Note by looking further down the list that χ *decreases* for surfaces that look *more complicated*. This is a good rule of thumb to keep in mind.

8. Polchinski's equation (1.2.31) is not the mathematical *definition* av χ: the integral over curvature he gives is related to the topological number χ (i topologi) by the Gauss-Bonnet theorem. You could probably reverse the logic and define χ from curvature, in which case Euler's topological definition would be a result. In my opinion it's more useful to obey the convention and call (1.2.31) the result of the Gauss-Bonnet theorem, in which each side is defined independently. In particular, some physics students are not familiar with the topological definition of χ for polyhedrons — indeed it doesn't seem very physical, talking about abstract surfaces with corners and edges. But such surfaces do occur in string theory, both in the so-called *moduli space* and in *orbifolds* in spacetime.

9. Even though the curvature at the point you pushed in does increase, so there is a positive addition to the integral of the curvature, there must be a negative contribution to the integral of the curvature that precisely cancels it. Indeed, the outer parts of your little indentation "bend in two different directions", like a saddle point, so they contribute negative curvature.

   In the smooth case this is fairly intuitive, but it is not so easy to apply directly to polyhedra. On the Wikipedia page about Euler characteristics, it looks like χ changes by "pushing in" part of a convex polyhedron. But then there are other edges, etc., so it is not a precise statement that those indentations alone "cause" the change in χ, as I was discussing above. Convex polyhedra by definition have no indentations of the kind I'm discussing here.

   (The theorem has far-reaching generalizations like the generalized Gauss-Bonnet theorem and the Riemann-Roch theorem. In the proof on this page there is an even more generalized version, the Grothendieck-Riemann-Roch theorem. But perhaps the greatest generalization is the index theorem. Curiously, *all* of these have some application in string theory, so please take a quick look at the Wikipedia pages, but don't expect to grasp most of it at this point.)

10. See solution.

11. The stabilizer group (little group) is what is left of a group of symmetries if you fix a "representative" element. For example, the little group for a fixed vector in three dimensions is rotations about that vector. In the same way, the little group in $D = 4$ for a fixed massless vector consists of rotations in the plane, $SO(2)$, which is $U(1)$. Those rotations are entirely spatial, that is why $SO(D-2)$ is relevant in generic dimension D. (At this point, Polchinski is *not* saying we have Wick rotated: if we did, we would from $SO(D-1,1)$ go to $SO(D)$, but there is no time then, and no light-like vector that can be nonzero.) $U(1)$ only has one generator, so at this point it sounds a little like a photon should then have only one possible state? However, quantum field theory has CPT symmetry, and for example a circularly polarized photon changes direction under parity P, so then there are two photon states, or $D-2$ for general dimensions. (This argument looks strange at first sight: in $D = 3$ there is apparently only one state, so how can it result from this "doubling" of states? However, parity is very different in odd and even dimensions: for example, in odd dimensions, $\det P = +1$, the same as for rotations.)

12. Yin's solution is clear.

Kapitel 2

1. *left-mover*, see (2.1.13a). Yes, it is against all logic that something moving to the right is called left-moving, but it is done here for historical reasons. Polchinski's "excuse" is that it depends on which direction you draw the σ^1 axis – not much of an excuse!

   If you really don't like left-moving waves that move to the right, you'd be welcome to switch $\sigma^1 \to -\sigma^1$, away from Polchinski's convention. But this is a little overkill: since we then immediately Wick rotate $\sigma^0 = -i\sigma^2$ to Riemannian signature $(++)$ and use complex coordinate $z = \sigma^1 + i\sigma^2$, any counterintuitive convention is quickly swept under the carpet, and we use a holomorphic (complex-analytic) function $f(z)$ to represent our original left-moving wave $f(\sigma^1 - \sigma^0)$. That goes to the right.

2. The operator $e^{i\phi(x)}$ that describes waves in collections of spins is a boson, even if the spins themselves are fermions. If $\phi(x)$ is an ordinary scalar field $\phi(z, \bar{z})$ in two dimensions, with Green's function[4] $\ln|z|^2$, we can reuse the calculations for X that has Green's function $(\alpha'/2)\ln|z|^2$ as in Polchinski, if we set $\alpha' = 2$. The conformal weight of $e^{ikX(z,\bar{z})}$ according to (2.4.17) is $\Delta = h + \bar{h} = \alpha' k^2/2$, so if $\alpha' = 2$ and $k = 1$ as in this case, then $\Delta = 2/2 = 1$. The *conformal spin* (difference of holomorphic and antiholomorphic weights) $h - \bar{h} = 0$ here, so you see from (2.1.13) that under $z \to e^{i\theta} z$, the operator $e^{i\phi(x)}$ rotates by a factor $(e^{i\theta})^{-h}((e^{i\theta})^*)^{-\bar{h}} = e^{i(h-\bar{h})\theta} = 1$, that is, not at all, as you might have expected from a spinless boson.[5]

   By contrast, if we take only "half" of $X(z, \bar{z}) = X_L(z) + X_R(\bar{z})$, for example $X_L(z)$ (a *chiral* part), then we only have $\Delta = h + 0 = 1/2$ and then rotation in the plane gives a factor $e^{i(h-\bar{h})\theta} = e^{i\theta/2}$, like a fermion, which we will come back and use in Ch. 10. We'll also talk more about bosonization later, an equivalence between a boson and two fermions. The relatively simple bosonization relevant here is special to 1+1 dimensions, but the idea that fermions can appear bound in pairs that act like bosons is of general interest, like in atomic nuclei, or in Cooper pairs in superconductivity. (You might think superconductivity is entirely unrelated to string theory, but see the student projects, or even Ed from 1984 [1].)

   This discussion will suffice for now to emphasize the intimate relation between string theory methods and stat-mech/cond-mat methods. In Ch. 7 we'll return to more cond-mat.

3. Yes, in general different symbols for normal ordering are needed, for example in (2.7.14). Of course if you consistently only use one of the various kinds there is no need to introduce several symbols, but there could be many confusing factors of 2 if you were to accidentally mix the various kinds without noticing. More to the point: operator normal ordering and functional-integral normal ordering are inherent to each formalism, and we need both formalisms.

4. Polchinski means that in Ch. 2 we have chosen unit gauge $\gamma_{ab} = \delta_{ab}$ (right away, see below (2.1.1)), then we no longer have diffeomorphism invariance. Einstein did not pick a form of the metric when arguing for invariance under general coordinate transformations! From this point of view, it is very surprising to find conformal invariance, and as Polchinski argues, it is certainly not present for generic forms of the action. A mass term does not itself transform under coordinate transformations of the worldsheet, that is correct. But the integral measure d^2z transforms, so having a mass term in the action S is not invariant under conformal transformations, even though the term in the Lagrangian is invariant. For a term in the action S to be conformally invariant, the term in the Lagrangian should cancel the transformation of the measure d^2z, as happens for the kinetic term of X.

[4] If you need a reminder about Green's functions, perhaps watch the video.

[5] Warning: the spin-statistics theorem familiar from 3+1 dimensions, that says that half-odd-integer spins are fermions and even-integer spins are bosons, needs to be generalized in two dimensions; in particular, there are other kinds of exchange symmetries than symmetry and antisymmetry. Another warning: later we will consider *ghosts*, whose defining characteristic also in four dimensions is that they have opposite statistics to what is dictated by the spin-statistics theorem.

This was the restrictive meaning here in Ch. 2. To contrast with e.g. Kiritsis, we need to see what Polchinski says when we do not set the metric to be flat, so there is a question about this in Ch. 3.

5. The coordinate σ^2 goes up in fig. 2.3a but radially outwards in fig. 2.3b (i.e. not just up in fig. 2.3b but for example also to the right).

   In Riemannian (i.e. Euclidean) signature $(++)$ it is of course not strictly correct to speak of "time" at all, since σ^2 is a spatial coordinate, that Polchinski calls "Euclidean time". Note that in the original meaning of σ^0 as time, it was certainly real, then by $\sigma^2 = i\sigma^0$, the new σ^2 should take imaginary values. But as is evident from writing $z = \sigma^1 + i\sigma^2$, this is not what is intended here: the space we are now considering is secretly an analytic continuation of the previous space: σ^2 is now viewed as real, so now the *original* σ^0 would have to be imaginary. It is then perhaps better to write $\sigma^0 \to -i\sigma^2$ than $\sigma^0 = -i\sigma^2$.

   While being pedantic, the expression "Euclidean time" is confusing if taken too literally, since there is no built-in causal structure in Euclidean (Riemannian) signature z or w that these figures refer to, i.e. there is no light-cone to keep objects following timelike trajectories, which is a defining property of "time". In more detail, the two-dimensional cone of lightlike vectors in (σ^0, σ^1) coordinates collapses to a point in the z coordinate, since in Riemannian signature, a vector of zero magnitude is really zero. The point of calling σ^2 "Euclidean time" despite not being time at all is just that we can *remember* there is causal structure when we revert to σ^0, when $\sigma^1 + i\sigma^2 = \sigma^1 - \sigma^0$: holomorphic functions become waves moving to the right, as discussed above, and in the (σ^0, σ^1) variables, no disturbance moves faster than the speed of light $v = 1$.

   At the risk of confusing the reader, contemplate the

6. The solution in Headrick is clear.

7. No, we showed that the worldsheet curvature scalar $R(z)$ is topological by itself, we did not allow it to multiply something that depends on z, like $X(z)$. We do reduce to that case if $X(z) =$ constant, so T_{ab} must only depend on X through derivatives ∂X, not directly. The term shifts c, so it does make a physical difference, and the weights h are also shifted. Even if you set the worldsheet curvature to zero locally there can be a global effect at $|z| = \infty$

   Å andra sidan blir termen topologisk om X är konstant, så T_{ab} måste bero på X enbart genom derivator ∂X, inte direkt. Termen ger en skiftad c, så det spelar roll, och vikterna h skiftas också. Även om man sätter världsytekrökningen till noll lokalt så kan det vara en global effekt i $|z| = \infty$ (nordpolen på Riemannsfären), som motsvarar en total derivata i T_{ab}. Med andra ord, en sfär är aldrig riktigt plan!

8. Det som händer är att med bakgrundsladdning Q får takyonen en förstaderivata-term ∂T i rörelseekvationen, och i Fourierrummet är ∂T komplex.

9. Det är ingen skillnad på Polchinskis algebra SL och Jürgens algebra $\mathfrak{sl}$. Det är vanligt i fysik att man betecknar både gruppen och algebran med SL, men det är förstås mindre förvirrande att ha olika symboler för de två ganska olika koncepten. (I synnerhet kan det finnas olika grupper som har samma algebra, vet du något exempel?)

Kapitel 3

1. En yta är icke-orienterbar om man inte överallt på ytan kan hitta ett (konsistent) fält av normalvektorer som pekar “utåt”. Eller ekvivalent: man kan inte välja *rotationsriktning kring normalvektorn* som är “moturs” , enligt högerhandsregeln. Möbiusbandet är ett välkänt exempel, i kapitel 7 kommer Klein-flaskan, som är ett annat, mer om det då. Både Möbiusbandet och Klein-flaskan har Eulerkarakteristik $\chi = 0$ så de kommer på enloopsnivå i strängteori, dvs. kapitel 7 i Polchinski, inte kapitel 6 som är trädnivå (sfär, skiva och projektivt plan).

2. Man skall kunna välja vilken metrik som helst, men för att konkret prata om att välja en så vill man ha en representant. Så man kan tänka sig att välja $g_{ab} = \hat{g}_{ab}$, sedan varierar man den $g_{ab} \to \hat{g}_{ab} + \delta g_{ab}$ och kräver att amplituder är invarianta. Det sista kommer i kap. 4. Med andra ord: man måste förankra beskrivningen i något, men beskrivningen beror inte på var man förankrar.

3. Poissons ekvation $\nabla^2\phi = \rho$ och entydighet för den givet randvillkor, som i Sturm-Liouville-problemet. (Dirichlet ger entydig lösning, men Neumann är bara entydig givet ett bevaringsvillkor. I strängteori har man periodiska randvillkor för slutna strängar, då är det inget problem med bevaring, men för öppna strängar måste man tänka efter lite mer.)

4. Konform transformation är en koordinattransformation (diffeomorfi) som bevarar metriken i formen $g_{ab} = \delta_{ab}$. För att det skall vara möjligt behöver man i allmänhet ha en kompenserande Weyltransformation. Det här kan vara ytterst förvirrande eftersom han satte metriken till $g_{ab} = \delta_{ab}$ i kapitel 2 och hade konform symmetri utan att verka göra någon kompenserande transformation. Men vi hade redan visat att Polyakov-verkan är invariant under Weyl-transformation, så man kan betrakta det som att det är inbyggt, så länge man skriver ut $\sqrt{g}g^{ab}$....

   (Jag kallade frågan “kulturaspekt”, för som han skriver betyder de här orden förvirrande nog olika saker för olika personer: i allmän relativitetsteori räknas en konform transformation i allmänhet inte som en diffeomorfi, utan som det som Polchinski kallar en Weyltransformation!)

5. Som sagt, det är icketrivialt. Det finns flera viktiga poänger, men två är: en spårlös symmetrisk har bara två oberoende komponenter, och kovariant derivata med avseende på $\bar{z}$ av en tensorkomponent $zz\cdots z$ är en vanlig derivata för det finns inga ickeförsvinnande “blandade” Christoffel-symboler att bygga upp en kovariant derivata med.

6. Det är totala c som måste vara noll, dvs. inklusive spöken, som det står i ekvation (3.4.23). Som vanligt i teoretisk fysik tillåter vi att introducera koncept som i någon bokstavlig bemärkelse är redundanta eller t.o.m. ofysikaliska (här inte Weyl-invarianta, trots att vi kräver att vår teori som helhet är Weyl-invariant), men som är praktiska att jobba med av olika anledningar, så länge det totala svaret kan garanteras vara fysikaliskt. (Det är ofta en intressant fråga huruvida det är nödvändigt eller ens en bra idé att göra så, dvs. om det inte skulle gå att alltid bara jobba med fysikaliska koncept. Tumregeln är att folk brukar prova det först, och gör man inte det så har någon märkt – kanske naivt – att det inte verkar gå så bra.). Standardexemplet är gaugeinvarians i elektromagnetism, eller vågfunktionen i kvantfysik. Ett aktuellt exempel att gå åt andra hållet, dvs. arbeta mer fysikaliskt, är kvantfältteori som är formulerad “på skalet” (*on-shell*, ett nyckelord där: *amplituhedron*).

7. Spökverkan är det man får från vakuumamplituden (0-punktsfunktionen). Däremot så vet vi ännu inget om banintegraler med externa tillstånd, som verkligen sprids (n-punktsfunktioner).

8. Man blir av med antisymmetriska tensorn och skalären (dilatonen) som man får från antisymmetriska respektiva spår-biten av sluten-sträng-tillståndet $\alpha^\mu\tilde{\alpha}^\nu|0;k\rangle$. Vi har inte diskuterat än vad de har för betydelse, så det verkar lika bra.

9. (3.7.12) är svaret på en mer allmän fråga, inte bara vad som händer i vakuumamplituden, utan man ber om Weylinvarians även med vertexoperatorer (störningar av vakuum, n-punktfunktion), som för given polarisering $\chi_{\mu\nu}$ inte är Lorentzinvariant (bara Lorentzkovariant). Det är kvant-effekter med kopplingskonstant $\alpha' \to 0$ i världsyteteorin. Termen β^{Φ} är däremot som förut.

10. Hela Einsteintenstorn; det går inte att läsa av från bara förekomsten av Riccitensorn att man inte har Einsteins ekvation. Man kan nämligen gå till "trace reversed form" av Einsteins ekvation genom att ta spåret av $\mathbf{G}_{\mu\nu} = \kappa \mathbf{T}_{\mu\nu}$ och lägga över det till högerledet, då har man $\mathbf{R}_{\mu\nu} = \kappa \tilde{\mathbf{T}}_{\mu\nu}$ för ett modifierat $\tilde{\mathbf{T}}_{\mu\nu}$. Man ser påståendet tydligare strax därefter, då Polchinski härleder det från en verkan, som man gör för Einstein ekvation.

11. X-CFT:n har $c = 25$, så man kan ersätta det med någon annan CFT som har $c = 25$, som då inte kommer att ha en tolkning som bosoniska strängar som rör sig i $D = 26$, utan t.ex. bosoniska strängar som rör sig i ett produktrum av fyra dimensioner samt några kompakta (ihoprullade) extradimensioner. Mer extremt kan man tänka sig att en del av teorin inte har någon geometrisk tolkning alls!

Kapitel 4

1. Definitionen av T^{ab} här är variation med avseende på metriken, så den klassiska rörelseekvationen för metriken är $T^{ab} = 0$. Gaugefixering betyder ju att "förankra" metriken till något $\hat{g}_{ab}$, då blir rörelseekvationen meningslös som dynamisk ekvation, istället blir den ett tvång, som vi har viss frihet i att implementera. Vi väljer det relativt svaga kravet (4.1.2). Man hade kunnat tänka sig att kräva det starkare $T^{ab} \equiv 0$ som operatorekvation, men det argumenterar han på nästa sida att det är så starkt att det inte blir några tillstånd kvar som uppfyller det, så man måste nöja sig med det svagare (4.1.2), och det visar sig räcka.

2. (Det här har lite karaktär av "chansning", vi provar det här, sedan det här, osv. Det är precis det som kallas OCQ ("old covariant quantization"). Metoden BRST som kommer sedan är den systematiska metoden.)

3. Påståendet är: det finns alltid ett variabelval som leder till att Hamiltonfunktionen $H = 0$ om man har invarians under tidsreparameteriseringar.

4. Ja, BRST har i sig inget att göra med kvantfysik.

5. Ja, det är lite överraskande. Man blev tvungen att ta till fermioniska variabler i Fadeev-Popov-DeWitt-metoden för att invertera determinanten. Det visar sig sedan att BRST-symmetrin är en slags "gömd" supersymmetri som relaterar fälten med spökena! Det är inte så otroligt överraskande när det gäller fria (icke-växelverkande) teorier som $X^\mu + bc$-CFT:n att den kanske är supersymmetrisk i någon bemärkelse, men det är mer överraskande att det verkar vara något liknande i Yang-Mills, som ju har ganska komplicerad växelverkan (pågående forskning).

6. Det står t.ex. i Schutz mattebok, eller i Polchinski Appendix B.2. Betrakta vektorpotentialen $A = A_\mu dx^\mu$ som en abstrakt enhet utan index, en 1-form. Då är $A \to A + df$ en gaugetransformation, där $df = \partial_\mu f dx^\mu$ för en funktion f (en "0-form"). Samtidigt har vi $F = dA$, där $F = F_{\mu\nu} dx^\mu \wedge dx^\nu$, och jag har infört den abstrakta antisymmetriska produkten $\wedge$ ("wedge"). A är alltså inte en sluten 1-form, för $dA \neq 0$, men F är automatiskt en sluten 2-form (varför?) och df är ett nolltillstånd som man kan addera utan att ändra det fysikaliska tillståndet. Kohomologin är fysikaliska vektorpotentialer, dvs. modulo gaugetransformationer df. Så efter en massa matematik är vi tillbaka till elektriska och magnetiska fält $\mathbf{E}$ och $\mathbf{B}$. (Notera att det inte är självklart i alla sammanhang att F är invariant under gaugetransformationer, t.ex. är $F^a_{\mu\nu}$ i ickeabelsk gaugeteori inte invariant, medan $F^a_{\mu\nu} F^{\mu\nu\, a}$ är det.)

Kapitel 5

1. I allmän relativitetsteori skulle man kunna tycka det verkar konstigt, men det är för att man ofta inte betraktar torusar där, som är kompakta ytor: periodiciteten påverkas av generiska koordinattransformationer.
   https://en.wikipedia.org/wiki/Teichm%C3%BCller_space
   Matematiker uttrycker det som att *Teichmüller-rummet för torusen är övre halvplanet* (Im $\tau > 0$). (Alla metriker med τ är inte heller sinsemellan olika, utan man måste dessutom för integrationen begränsa τ till ett "lagom" stort område, fundamentalregionen fig. 5.2, för att inte räkna samma torus flera gånger. Begränsningen från hela komplexa planet τ till övre halvplanet till fundamentalregionen är bägge relativt "små" begränsningar, dvs. först hade vi oändligt många parametrar (3 funktioner, dvs. $3 \cdot \infty$ parametrar), efter diff $\times$ Weyl-gaugefixering har vi bara en futtig komplex parameter τ, det är en "stor" begränsning. I det sista steget begränsar vi bara *värdena* som τ kan anta lite grand. Men även den sista begränsningen visar sig vara väldigt viktig i strängteori, som vi ser i kapitel 7.)

2. Ja, enligt reglerna för Grassmann-integraler så är $\int [db] = 0$ men $\int [db] b = 1$.

3. Insertion av T genererar koordinattransformationer (2.3.15). Jämför också (7.2.13).

4. Att resultat är en total derivata, är ytterligare en anledning varför man skall integrera över moduli t^k, för annars är resultatet alltså inte BRST-invariant.

5. A priori är positionerna för vertexoperatorer och metrikmoduli oberoende, men det är konceptuellt viktigt att det går att fixera alla vertexoperator och låta de positionerna vara inbyggda i överlappfunktioner (*transition functions*, minns definitionen av mångfald). Överlappsparametrar är inte riktigt samma sak som metrikmoduli men konceptuellt känns det mer likt, mer geometriskt än vertexoperatorpositionerna. (Om man tänker efter ännu mer är vertexoperatorpositionerna också "geometriska", så i någon bemärkelse var det uppenbart att det skulle finnas någon relation, frågan var inte om utan hur.)

Kapitel 6

1. Nej, r är sfärradien, en konstant, inte radien i polära koordinater.

2. Möbiusbandet och Klein-flaskan har bägge Eulerkarakteristik $\chi = 2 - 2g - b - c = 0$ (s.101) så de kommer på enloopsnivå i strängteori (g_s^0 i taylorutvecklingen av amplituder, jämfört med nollte ordningens term g_s^{-2}) dvs. kapitel 7.

3. $Tr(t_1^a[t^{a_2}, t^{a_3}]) \propto f^{abc}$. Eftersom strukturkonstanten f^{abc} förekommer i 3-punkt-vertexet och rörelsemängder förekommer i differenser som k_{12} är det klart att när man kontraherar med tre externa polariseringar så får man något som de första tre termerna i (6.5.15). Men inte termen med α'! Och det är ju som det skall vara, den försvinner då $\alpha' \to 0$.

Kapitel 7

1. Teichmüllerparametern τ är en redundant parameter i bemärkelsen att vi integrerar över den (annars är amplituden inte BRST-invariant). Med andra ord, diff $\times$ Weyl-invarians räcker för att få metriken till formen (5.1.9) med τ, men inte för att sätta t.ex. $\tau = i$ så $g_{ab} = \delta_{ab}$.

2. Bara konstanter är holomorfa vektorfält på torusen.

3. Nej, "*figure 8 immersion*" skär också sig själv i en cirkel, men det syns inte så tydligt i bilderna. Man brukar inte diskutera den varianten i strängteori.

4. Faktorn $2\pi/\partial_\nu\vartheta_1$ bestäms inte av (7.2.1) när den inte beror på w. Lösningen är i uppgift 7.1.

5. Bara 9 av 30 potenser överlever: exponenterna $k(3k-1)/2 = 0, 1, 2, 5, 7, 12, 15, 22, 26$, de s.k. pentagonala talen. De fantastiska kancelleringarna fångas av en klassisk formel (Euler, förstås). Generaliseringen till thetafunktioner är sedan relativt enkel och gjordes av Jacobi. Beviset av ekvivalensen beskrivs t.ex. på
   https://en.wikipedia.org/wiki/Jacobi_triple_product
   Det står mer om sådana här tal t.ex. i Ginspargs "Applied Conformal Field Theory" [2]. Jacobis formel följer också från Weyl-Kac-karaktärsformeln i specialfallet $SU(2)$, som i Jürgens andra bok [**?**], sektion 2.6.

Kapitel 8

1. $\chi = 24$ för alla T^4/Z_N.

2. Blåser man upp de singulära punkterna (ersätter dem med Eguchi-Hanson-rum, eller ALE-rum) får man K3, en glatt Calabi-Yau-mångfald med fyra reella dimensioner som har $\chi = 24$.

3. Det här har ordentliga fördjupningar, som Satake-Euler-karakteristiken för orbifalder, och holomorfa Lefschetz-satsen [3]

$$\chi^\theta(M, V) = \int_{M^\theta} \frac{\mathrm{ch}_\theta(\mathrm{V})\mathrm{Td}(\mathrm{TM}^\theta)}{\mathrm{ch}_\theta(\Lambda_{-1}\bar{\mathrm{N}}_{\mathrm{M}^\theta})} \tag{1}$$

   som jag inte förstår särskilt bra.

4. Det är olika $\tilde{\ell} = 1/\ell$. Notationen är koncis men förvirrande.

5. Högerledet i (8.2.21) är tecknet på skillnaden i rumsliga delen σ^1 av världsytekoordinaten $z = \sigma^1 + i\sigma^2$, dvs. vilken av de två operatorna X_L som är "mer till vänster" på världsytan. (Den frågan är bara meningsfull eftersom han kräver samma världsytetid σ_2 för att definiera kommutatorerna.)

Chapter 10

1. $\nu = 0$ for periodic, is called Ramond (R), $\nu = 1/2$ för antiperiodic, is called Neveu-Schwarz (NS). Ramond's 1971 paper was shortly before the NS paper, the history papers are 1401.5977 (Ramond) and 1201.0981 (Schwarz[6]). Polchinski's argument is that minus signs are allowed for fermions since observables (and terms in the Lagrangian) have an even number of fermions. (In orbifolds, this restriction is relaxed and even bosons can have minus signs, but this is in 10 dimensions with no orbifold.)

2. The sum runs over "integers + ν", not really over the integers themselves. I'm not saying it's *wrong* as written, since the set of numbers that are being summed over can be labelled by integers even if the summation variable itself is not integer. But at first sight I found this slightly confusing, so maybe you did too. It is stated more clearly a little later, though.

3. There's a one-half from the transformation $w \to z$, and since $\nu = 0$ in the Ramond sector, ψ is two-valued there, i.e. $\psi_R(e^{2\pi i} z) = -\psi_R(z)$ (a monodromy). It would create problems if this were to make amplitudes multivalued in some sense. (In conformal field theory, non-integer powers of z in the OPE are called "non-local", since the branch cuts are extended in the z plane.) However, as we will see, string amplitudes still aren't multi-valued: physical vertex operators always have some extra factor that compensates non-integer powers of z so the total has integer powers.

4. It's the domain of the fermion field that is doubled, from just the "physical" domain to the "image" domain. Compare the method of image charges in electrostatics. The important conceptual idea is that although the boundary condition for open strings relates $\psi(z)$ and $\tilde{\psi}(\bar{z})$, we can get away with just a single holomorphic "doubled" field $\psi(z)$, and avoid working directly with $\tilde{\psi}(\bar{z})$. The price of this is that $\psi(z)$ will be multivalued in z.

5. According to Ch.2, the conformal weight of the nonholomorphic operator $e^{ip\cdot X(z,\bar{z})}$ is $(h,\bar{h}) = (\alpha' p^2/4, \alpha' p^2/4)$, cf. (2.2.13) och (2.4.17). So the total conformal weight is $\Delta = h + \bar{h} = \alpha' p^2/2$. But note that the contraction (2.1.21b) is $-(\alpha'/2)\ln|z|^2 = -(\alpha'/2)(\ln z + \ln \bar{z})$.

   The weight of a holomorphic (left-moving) field $e^{ip\cdot X_L(z)}$ is then given by the left half $(h,\bar{h}) = (\alpha' p^2/4, 0)$, but to get canonical normalization for $H(z)$ in $e^{ipH(z)}$, we should set $\alpha' = 2$, so $h = 2p^2/4 = p^2/2$. (Note that this just "corresponds": as Polchinski emphasizes, this H has no direct relation to the coordinate X, so this "setting α' to some value" should not be confused with setting the actual α' to some value, we simply postulate $H(z)H(0) \sim -\ln z$, and we can then recycle the results for X if we set $\alpha' = 2$ there.) This fits with the weight of $e^{i(\nu-1/2)H}$ being $h = (\nu - 1/2)^2/2$. (To emphasize that some operator has $h \neq \bar{h}$ you can talk about the *conformal spin* $h - \bar{h}$ in addition to the conformal weight $\Delta = h + \bar{h}$. The conformal spin of a holomorphic operator is $h - 0 = h$.) So for integer p, the worldsheet spin is half-integer.

   But yuck, doesn't that mean that $\Theta \sim e^{\pm\frac{1}{2}iH}$ has conformal weight (and spin) $(1/2)^2/2 = 1/8$, that sounds weird? Yes, but we don't use $e^{\pm\frac{1}{2}iH}$ by itself, there are five different H's in our total spin field Θ, and together they have conformal weight $5/8$. Which probably doesn't sound much better, but then with the ghost $e^{-\phi/2}$ giving $3/8$ (ϕ has background charge so the naive weight $(1/2)^2/2 = 1/8$ gets an extra contribution) so we get $h = 5/8 + 3/8 = 1$ for the physical[7] vertex operator $e^{-\phi/2}\Theta$.

6. Spectral flow: as he writes, ν and $\nu + 1$ are the same when it comes to periodicity, but not for the ground state. Two degenerate ground states $s = \pm 1/2$ for a fermion in the R sector

[6]Here with hindsight, Schwarz calls Ramond's paper "a very inspired and important development", but in the actual NS paper not much credit is given to Ramond. Of course, these developments happened very rapidly and at the time it must not have been obvious what was important. In fact, it still isn't obvious what aspects of supersymmetry are important for real-world physics.

[7]For historical completeness, it's fun to notice that in the Green-Schwarz-Witten book, this is phrased as "future work".

seems reasonable. So you can imagine switching with an integer step down: $\nu = -1/2$ instead of $\nu = 1/2$ in the NS sector, which seems to fit with $\psi \sim e^{iH}$. I don't know an independent argument why we should go from the conventional $\nu = 1/2$ to $\nu = -1/2$, except that otherwise we get the wrong answer.

7. **Spin-statistics theorem**. Our old friends the coordinates ∂X^μ are worldsheet bosons (conformal spin 1, see above) and spacetime bosons (vectors, spin 1). So that doesn't seem strange. But ψ^μ is a worldsheet fermion (conformal spin 1/2, see above) and spacetime boson (vector, spin 1), and the same holds for $e^{-\phi}$: worldsheet fermion (conformal spin 1/2) and spacetime boson (scalar, spin 0). So in the superstring there seems to be no connection between spin and statistics on the worldsheet (where we construct our theory) and in spacetime (where we think we already understand the spin-statistics theorem). But, we can form a composite operator. So $: e^{-\phi}\psi^\mu : (z)$ normal-ordered at the point z is both worldsheet boson and spacetime boson. That feels a little nicer. But that it's not obvious that any of this is a problem when we have ghosts, since already in spacetime quantum field theory, ghosts violate the spin-statistics theorem. So do b and c in the bosonic string theory: they have integer conformal spin but are fermions. In some textbooks one gets the impression that in quantum field theory, ghosts cannot occur as external states, but that's not really true, and in fact that is what we want to allow when we construct vertex operators (see video 6 about ghosts). Bonus question about ghosts: the ghosts β och γ in superstring theory are bosonic, so how come $e^{-\phi}$ is fermionic?

Kapitel 12

1. Den ena är inte sluten form.
2. Krökningstermer. Det syftar på superfältsigmamodellen i kapitel 12, då sådana termer uppstår som Riemanntensorn kontraherad med 4 fermioner.
3. Konforma dimensionen skiljer sig med 1/2.
4. Standardmodellen är anomalifri, men inte om man tar bort toppkvarken.
5. Ja, det är en term av ordning $\alpha' k^3$ i 3-punkts-amplituden för gaugebosoner i bosonisk strängteori.

Kapitel 13

1. Ja, det är samma McGreevy.
2. Det är i strängramen (*string frame*). I Einsteinramen (*Einstein frame*) så ändras spänningen.
3. Polchinski skriver att den är helt bestämd av supersymmetri. Men det är inte jätteviktigt om man bara vill räkna antalet fält och deras kvanttillstånd, som vi vill.

Kapitel 14

1. Det är $\kappa^2 = 8\pi G_N$ i gravitationslagen (3.7.26) och i allmän relativitetsteori sätter man ofta Newtons konstant $G_N = 1$, så $\kappa^2 A/4 = 8\pi A/4 = 2\pi A$.

Appendix B

Jo, det gör det, det är bara en praktisk organisering att "koppla ihop notation", stegoperatorer är egentligen inte direkt kopplat till kvantfysik, även om det är där vi använder det.

Ja, det är allmänt med periodiciteten 8:
`https://en.wikipedia.org/wiki/Bott_periodicity_theorem`

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