Problem Set 2, String Phenomenology

May 21, 2008

1 Intersecting Branes and the MSSM

a) Derive the relation

$$\tan\phi = \frac{p}{q}\frac{R_1}{R_2 \sin\theta} + \cot\theta$$

for a brane with wrapping numbers (p, q) at an angle ϕ from the y axis on a 2-torus where U makes an angle θ with the x axis. This should be very easy.

b) The intersection number of two 1-cycles Π_a and Π_b on a torus is $I_{ab} = p_a q_b - q_a p_b$. The image cycle of a cycle Π_a under the orientifold action ΩR is called $\Pi_{a'}$, and on a square torus, ΩR acts as $(p_a, q_b) \to (-p_a, q_b)$. If the number of generations is $N = I_{ab} - I_{ab'}$, show that naively, you cannot get three generations. Taking the discussion in class into consideration, what is the resolution?

c) Consider one stack of 2 D6-branes giving $SU(2)_{\text{ew}}$ of the MSSM wrapping the 3-cycle $\Pi_1 = (q_1^{(1)}, p_1^{(1)}) \times (q_1^{(2)}, p_1^{(2)}) \times (q_1^{(3)}, \tilde{p}_1^{(3)}) = (1, 0) \times (1, -1) \times (1, 3/2)$ in T^6, and one stack of 3 D-branes giving $SU(3)_{\text{color}}$ wrapping the 3-cycle $\Pi_2 = (1, -1) \times (1, 0) \times (1, 1/2)$ in the same notation. Here $U_1^{(1)} = U_1^{(2)} = 0$ and $U_1^{(3)} = 1/2$. (U_1 denotes the real part of U, and the superscripts (1), (2), (3) label which T^2 in $T^2 \times T^2 \times T^2$). Using the formula from a) and the discussion in class, show that the electroweak and color branes are each supersymmetric.

d) For the ambitious, show that the model in `hep-th/0107143` (wrapping numbers in Table I) satisfies the tadpole constraints (eqs. (2)). This will give you a feeling for how hard it is to satisfy them (try changing the wrapping numbers!), leading to in this case a fairly unique semirealistic model.

2 String correction to Kähler metrics of matter fields

Calculate the generic modulus-dependent integral

$$\int_0^\infty d\ell \, \frac{\vartheta_1'(ia, 2i\ell)}{\vartheta_1(ia, 2i\ell)}$$

(i.e. a is a parameter that can depend on spacetime moduli, but is constant with respect to the ℓ integral) using the following little toolbox:

$$\frac{\nu\vartheta_1'(0,\tau)}{\vartheta_1(\nu,\tau)} = \exp\left(\sum_{k=1}^\infty 2\zeta(2k)E_{2k}(\tau)\frac{\nu^{2k}}{2k}\right)$$

$$\int_0^\infty d\ell \, (E_{2n}(2i\ell) - 1) = \frac{2^{-2n}\pi^{1-2n}(2n)!}{(1-2n)|B_{2n}|}\zeta(2n-1) \qquad (n > 1)$$

where ζ is the Riemann zeta function, and B_{2n} is a Bernoulli number. ($E_{2k}(\tau)$ is an Eisenstein series, but you will not need to know that.) Just like last time, discard divergences (that will cancel when we add different contributions of this kind). In particular, as you can see from taking the $n \to 1^+$ limit, the $n = 1$ case above is divergent, so discard it. *Hint*: To get the answer in a nice closed form, i.e. to perform the infinite sums, you will need one more formula from Gradshteyn and Ryzhik (or your favorite source of correct formulas).