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What happens when an accelerating observer detects a Rindler particle

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The nature of the interaction between a quantum field and an accelerating particle detector is analyzed from the point of view of an inertial observer. It is shown in detail for the simple case of a two-level detector how absorption of a Rindler particle corresponds to emission of a Minkowski particle. Several apparently paradoxical aspects of this process related to causality and energy conservation are discussed and resolved.

I. INTRODUCTION

It is by now well known that for a free quantum field in its vacuum state in Minkowski spacetime, an observer with uniform acceleration a will feel that he is bathed by a thermal distribution of quanta of the field at temperature T given by[1] $kT=\hbar a/2\pi c$. In particular, an accelerating particle detector will be excited by the quantum field. For a simple particle detector consisting of a quantum-mechanical system coupled linearly to the field this effect has been investigated previously.[1] The main purpose of this paper is to investigate in detail the influence of such a detector on the state of the quantum field, thereby enabling us to obtain an "inertial interpretation" of what the accelerating observer would view as the absorption of a quantum by the detector.

Our investigation is motivated by the following considerations. The primary concern is to examine the "reality" of acceleration radiation. From the inertial viewpoint, the quantum field is in its vacuum state; no particles are present. Nevertheless, an inertial observer must agree that an accelerating particle detector will get excited. In some sense, then, the acceleration radiation becomes "real" to an inertial observer by virtue of its interaction with the particle detector. Precisely how does an inertial observer interpret the act of absorption of a Rindler particle? What is the resultant state of the quantum field after such an absorption has occurred?

As we shall show below, the inertial observer interprets the absorption of a Rindler particle as the emission of a Minkowski particle. However, this leads to the following apparent paradoxes which also comprise part of the motivation for our investigation. First, the Minkowski particle has a substantial probability to be found in a region of spacetime noncausally related to the "Rindler wedge" containing the accelerating particle detector. Consequently, it might appear that the decision to turn on a particle detector in one Rindler wedge would result in an increase in the expected stress-energy in the other Rindler wedge. Thus, we appear to have a mechanism for violating causality. As we shall see below, this difficulty is resolved when higher-order processes are taken into account. Second, there is an apparent conflict between the determinations by the inertial and accelerating observers as to whether energy has been gained or lost by the quantum field during the process of detection. According to the inertial observer, when a particle is emitted the expected stress-energy tensor of the quantum field satisfies the usual positive-energy condition, and thus the expected "boost energy" of the field in a Rindler wedge must increase. (Here the boost energy is defined by $E=\int \langle T_{ab}\rangle b^a dS^b$, where the integral is taken over a Cauchy surface S for the Rindler wedge and b^a is the boost Killing field.) However, according to the accelerating observer, a field quantum has been absorbed and hence it would seem that the expected field energy as determined by this observer (i.e., the expected boost energy) in his Rindler wedge goes down. As we shall see in Sec. III, this apparent discrepancy is resolved by the fact that, quite generally, the absorption of a particle by a detector placed in a thermal bath *increases* the energy of the quantum field.

In Sec. II we shall review the demonstration that the Minkowski vacuum state corresponds to a thermal state in Rindler spacetime. Our simple model of a particle detector will be given in Sec. III, and its behavior in a thermal bath will be studied. Finally, in Sec. IV, the behavior of such a detector as an emitter when accelerating in Minkowski spacetime will be analyzed.

Although we shall consider here only quantum fields in Minkowski spacetime, similar phenomena occur in de Sitter spacetime[2] and may be of significance for understanding the behavior of a quantum field interacting with matter (or with itself) during an inflationary phase of expansion.

II. RINDLER DESCRIPTION OF THE MINKOWSKI VACUUM

In this section, we shall briefly outline the derivation of the well-known relation between the accelerating and inertial descriptions of free quantum fields in flat spacetime.[1] For definiteness, we shall treat the case of a real Klein-Gordon scalar quantum field in four-dimensional Minkowski spacetime. However, the validity of our results for other free fields will be manifest from the fact that we shall not use any properties specific to the Klein-Gordon field in our analysis. We shall employ units with $\hbar = c = 1$ throughout the paper and will use metric signature $-+++$.

In addition to the usual time-translation Killing vector fields, Minkowski spacetime also possesses boost Killing fields which are timelike in portions of the spacetime. In terms of a global inertial coordinate system t,x,y,z, the Killing field which generates a boost about the origin in the x direction is

$$b^a = a(xt^a + tx^a) , \tag{2.1}$$

where a is a constant. Thus, b^a is timelike in the two Rindler wedges I and II of Fig. 1. In these regions, the proper acceleration of an orbit of b^a is

$$\begin{aligned} A^a &= \frac{1}{V^2} b^c \nabla_c b^a \\ &= \nabla^a \ln V , \end{aligned} \tag{2.2}$$

where $V^2 = -b^a b_a = a^2(x^2 - t^2)$. Thus, the magnitude of the proper acceleration is

$$A = (A^a A_a)^{1/2} = (x^2 - t^2)^{-1/2} = a/V . \tag{2.3}$$

Thus, a is just the acceleration of the orbit along which $V=1$, i.e., along which Killing time agrees with proper time.

The Rindler wedge I—viewed as a spacetime in its own right—is globally hyperbolic and possesses the static, everywhere-timelike Killing vector field b^a. Hence, one may apply the general procedures for defining a quantum field theory in a static, globally hyperbolic spacetime to this Rindler wedge.[3] Similarly, Rindler wedge II also is a static, globally hyperbolic spacetime. Most of the remainder of this section is devoted to reviewing the relation between the description of the quantum field in regions I and II obtained by this "Rindler quantization" procedure to that originally obtained by the standard Minkowski quantization prescription for all of Minkowski spacetime.

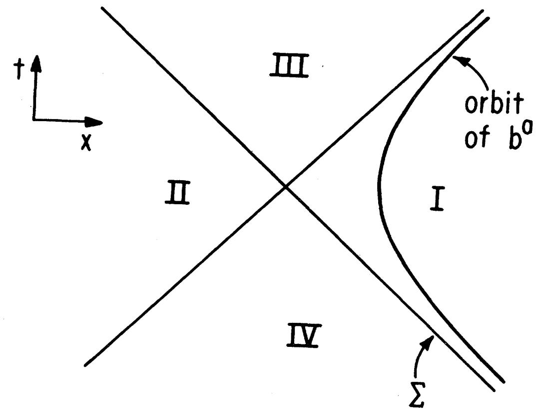

FIG. 1. A spacetime diagram of Minkowski spacetime. In the "Rindler wedges" I and II, the boost Killing field b^a is timelike.

The equation satisfied by the Klein-Gordon field in Minkowski spacetime is

$$(\Box - m^2)\Phi = 0 . \tag{2.4}$$

The space of solutions to this equation has an inner product defined on it by

$$\langle \phi_1, \phi_2 \rangle = \frac{i}{2} \int_\Sigma [\phi_1^* \partial_\mu \phi_2 - (\partial_\mu \phi_1^*)\phi_2] dS^\mu , \tag{2.5}$$

where Σ is some three-dimensional Cauchy surface for the spacetime and dS^μ is the surface element for that three-surface. This inner product is nonpositive definite but becomes positive definite when restricted to an appropriate subspace of solutions.

In the Minkowski prescription, the field operator is given by

$$\Phi = \sum_i [F_i a_M(F_i) + F_i^* a_M^\dagger(F_i)] , \tag{2.6}$$

where $\{F_i\}$ comprise an orthonormal (in the Klein-Gordon inner product) basis of Minkowski positive-frequency solutions of the Klein-Gordon equation and $a_M(F_i)$ and $a_M^\dagger(F_i)$ are the corresponding annihilation and creation operators for these states on the symmetric Fock space constructed from the Hilbert space of Minkowski positive-frequency solutions. Here "Minkowski positive frequency" is defined by means of Fourier transforms with respect to inertial time t, i.e., a time parameter derived from an ordinary time-translation Killing field. The Minkowski annihilation and creation operators can be expressed in terms of Φ by

$$\begin{aligned} a_M(F_i) &= \langle F_i, \Phi \rangle , \\ a_M^\dagger(F_i) &= -\langle F_i^*, \Phi \rangle . \end{aligned} \tag{2.7}$$

On the other hand, in the Rindler description, we have

$$\Phi = \begin{cases} \sum_j [\alpha_j a_{RI}(\alpha_j) + \alpha_j^* a_{RI}^\dagger(\alpha_j)] & \text{region I} \\ \sum_j [\beta_j a_{RII}(\beta_j) + \beta_j^* a_{RII}^\dagger(\beta_j)] & \text{region II} . \end{cases} \tag{2.8}$$

Here $\{\alpha_j\}$ is an orthonormal basis of Rindler positive-frequency solutions in region I and $a_{R\text{I}}(\alpha_j)$ and $a^\dagger_{R\text{I}}(\alpha_j)$ are the corresponding annihilation and creation operators, while $\{\beta_j\}$, $a_{R\text{II}}(\beta_j)$, and $a^\dagger_{R\text{II}}(\beta_j)$ are the analogous quantities for region II. Note, however, that the relevant Killing vector field is b^a, so we use "Rindler positive frequency" defined by Fourier transforms with respect to "Rindler time" τ (i.e., parameter along b^a) rather than inertial time t.

By equating the two expressions (2.6) and (2.8) for Φ, we obtain in the usual manner the Bogoliubov transformation relating the Rindler and Minkowski annihilation and creation operators. We find

$$a_M(F)=a_{R\text{I}}(\phi_\text{I}^+)-a^\dagger_{R\text{I}}(\phi_\text{I}^{-*})+a_{R\text{II}}(\phi_\text{II}^+)-a^\dagger_{R\text{II}}(\phi_\text{II}^{-*})\ . \tag{2.9}$$

Here F denotes an arbitrary one-particle Minkowski state, i.e., a positive-frequency (with respect to *inertial* time) solution of the Klein-Gordon equation, and $a_M(F)$ is the Minkowski annihilation operator associated with that state. The quantity ϕ_I^+ denotes the one-particle Rindler state in region I obtained by restricting the function F to region I and then taking its Rindler positive-frequency part, while $a_{R\text{I}}(\phi_\text{I}^+)$ is the corresponding Rindler annihilation operator. Similarly ϕ_I^- denotes the (Rindler time) negative-frequency part of F in region I and $a^\dagger_{R\text{I}}(\phi_\text{I}^{-*})$ is the creation operator of ϕ_I^{-*}. The quantities for Rindler wedge II are defined similarly.

The states of the quantum field can be characterized either as Minkowski particle states or as Rindler particle states. The relation between these two characterizations follows directly from Eq. (2.9). To determine this relation explicitly, we must explicitly find the Rindler-time positive- and negative-frequency parts of a sufficiently wide class of inertial-time positive-frequency solutions.

This task can be accomplished as follows. We introduce null coordinates u,v by

$$u=t-x\ , \tag{2.10}$$

$$v=t+x\ . \tag{2.11}$$

In terms of the coordinate vector fields of this coordinate system, the boost Killing field b^a is given by

$$b^a=a\left[v\left[\frac{\partial}{\partial v}\right]^a-u\left[\frac{\partial}{\partial u}\right]^a\right]\ . \tag{2.12}$$

Consider, now, the three-dimensional null plane Σ defined by $v=0$ (see Fig. 1). On Σ, the boost Killing field b^a is normal to Σ and parallel to the null translational Killing field $(\partial/\partial u)^a$. The relation between "boost time" τ and "inertial time" u on Σ is given by

$$\left[\frac{\partial}{\partial \tau}\right]^a=-au\left[\frac{\partial}{\partial u}\right]^a=-a\left[\frac{\partial}{\partial \ln|u|}\right]^a, \tag{2.13}$$

which implies

$$\tau=-\frac{1}{a}\ln|u|\ . \tag{2.14}$$

It is well known that Σ is *not* a Cauchy surface for Minkowski spacetime and that for the problem of the classical evolution of a Klein-Gordon field, specification of the value of the field on Σ does not suffice to uniquely determine a solution. However, the only obvious types of solutions which can fail to "register" on Σ are (superpositions of) plane waves (in the massless case only) having propagation vectors exactly in the negative x direction. However, such solutions do not have appropriate asymptotic falloff properties at infinity. This suggests that a solution which is well behaved at infinity is uniquely determined by its value on Σ. In the massless case, this idea can be formulated precisely and proven by examining the conformal completion of Minkowski spacetime.[4] In conformally completed Minkowski spacetime, all the null geodesic generators of Σ begin at a single point p of $\mathscr{I}^-$ and end at a single point q of $\mathscr{I}^+$. Indeed, Σ comprises the entire future light cone of p except for one null geodesic generator on $\mathscr{I}^-$, and it comprises the past light cone of q except for one generator on $\mathscr{I}^+$. Now, if we take as a precise asymptotic falloff condition that the conformally weighted Klein-Gordon field be smooth everywhere on $\mathscr{I}^-$ and $\mathscr{I}^+$, then its value on these remaining two generators will be determined from its value on Σ by continuity. But data on the future light cone of p and past light cone of q *do* suffice to determine the field throughout Minkowski spacetime.[5] Thus, any solution of the massless Klein-Gordon equation in Minkowski spacetime which is well behaved at null infinity is uniquely characterized by its value on Σ. This proof breaks down in the massive case since the conformally transformed equation no longer is smooth on $\mathscr{I}^-$ and $\mathscr{I}^+$, but we shall assume that asymptotically well-behaved solutions also are determined by data on Σ in this case.

Now the solutions which are positive frequency with respect to inertial time are precisely those whose data on Σ are positive frequency with respect to u, whereas the solutions which are positive frequency with respect to Rindler time in region I are those whose value on the portion of Σ with $u<0$ is positive frequency with respect to $\tau=-(1/a)\ln|u|$. Similarly the solutions which are positive frequency with respect to Rindler time in region II are those whose value on the $u>0$ portion of Σ are positive frequency with respect to $\tau=+(1/a)\ln|u|$. (The change in sign of τ for region II compensates for the fact that b^a is past-directed timelike there.)

Let $\phi_{\text{I}\omega}$ be a solution to the Klein-Gordon equation which on Σ is given by

$$\phi_{\text{I}\omega}=\begin{cases}\phi(y,z)\,e^{-i\omega\tau}=\phi(y,z)\,e^{(i\omega/a)\ln(-u)}, & u<0\\ 0, & u>0\ ,\end{cases} \tag{2.15}$$

where $\phi(y,z)$ is an arbitrary function of y and z and $\omega>0$. Then $\phi_{\text{I}\omega}$ gives rise to a purely positive-frequency solution (with respect to Rindler time) in region I. (Note that $\phi_{\text{I}\omega}$ is not well behaved at $u=0$, does not go to zero as $u\to-\infty$, and consequently is not normalizable. However, this is easily remedied by constructing wave packets in the manner described in Ref. 6 and the reader should interpret $\phi_{\text{I}\omega}$ below as being any wave packet with frequencies peaked sharply about ω.) Similarly, we define $\phi_{\text{II}\omega}$ on Σ to be the complex conjugate of the "time reverse" of $\phi_{\text{I}\omega}$,

$$\phi_{\mathrm{II}\omega}=\begin{cases}0, & u<0\\ \phi^*(y,z)\,e^{-(i\omega/a)\ln u}, & u>0\end{cases} \tag{2.16}$$

so that $\phi_{\mathrm{II}\omega}$ gives rise to a purely positive-Rindler-frequency solution in Rindler region II. The key observation needed to relate the inertial and Rindler notions of positive-frequency solutions is that the function

$$F_{1\omega}=(\phi_{\mathrm{I}\omega}+e^{-\pi\omega/a}\phi^*_{\mathrm{II}\omega})/(1-e^{-2\pi\omega/a})^{1/2} \tag{2.17}$$

on Σ is purely positive frequency with respect to u and thus gives rise to a solution which is purely positive frequency with respect to inertial time.[1,6] (Note that although this is strictly true only if $\phi_{\mathrm{I}\omega}$ and $\phi_{\mathrm{II}\omega}$ are exact eigenfunctions of $\partial/\partial\tau$, it will also be approximately true for functions which are normalizable wave packets of τ whose Fourier transform is sharply peaked around ω.) Thus, by Eq. (2.9), for all $\phi_{\mathrm{I}\omega}$ of the form (2.15), we have

$$a_M(F_{1\omega})=[a_{R\mathrm{I}}(\phi_{\mathrm{I}\omega})-e^{-\pi\omega/a}a^\dagger_{R\mathrm{II}}(\phi_{\mathrm{II}\omega})]/(1-e^{-2\pi\omega/a})^{1/2}\,. \tag{2.18}$$

Similarly, we find that the solution $F_{2\omega}$ defined on Σ by

$$F_{2\omega}(u)=F^*_{1\omega}(-u) \tag{2.19}$$

also is purely positive frequency and hence

$$a_M(F_{2\omega})=[a_{R\mathrm{II}}(\phi_{\mathrm{II}\omega})-e^{-\pi\omega/a}a^\dagger_{R\mathrm{I}}(\phi_{\mathrm{I}\omega})]/(1-e^{-2\pi\omega/a})^{1/2}\,. \tag{2.20}$$

The Minkowski vacuum $|0_M\rangle$ satisfies $a_M(F_{1\omega})|0_M\rangle=a_M(F_{2\omega})|0_M\rangle=0$. By applying Eqs. (2.18) and (2.20) to $|0_M\rangle$ one obtains the following expression for $|0_M\rangle$ as a Rindler state:[1]

$$|0_M\rangle=\prod_j\left[N_j\sum_{n_j}e^{-\pi n_j\omega_j/a}|n_j,\mathrm{I}\rangle\otimes|n_j,\mathrm{II}\rangle\right], \tag{2.21}$$

where $N_j=(1-e^{-2\pi\omega_j/a})^{1/2}$. Here the product is taken over a complete set of Rindler modes j of the forms (2.15) and (2.16) and $|n_j,\mathrm{I}\rangle$ denotes the state with n_j particles in mode $(\phi_{\mathrm{I}\omega})_j$ in region I, while $|n_j,\mathrm{II}\rangle$ denotes the state of n_j particles in mode $(\phi_{\mathrm{II}\omega})_j$ in region II. Note that there are very strong correlations between region I and region II. If an observer in region I determines that n_j particles in mode $(\phi_{\mathrm{I}\omega})_j$ are present, then with unit probability, an observer in region II will find n_j particles in mode $(\phi_{\mathrm{II}\omega})_j$.

By "tracing out" over the degrees of freedom associated with Rindler region II, we obtain the density matrix ρ for region I given by

$$\rho=\prod_j\left[N_j{}^2\sum_{n_j}e^{-2\pi n_j\omega_j/a}|n_j,\mathrm{I}\rangle\otimes\langle n_j,\mathrm{I}|\right]. \tag{2.22}$$

This is precisely a thermal density matrix. Thus, we find that the Minkowski vacuum corresponds to a thermal state in each Rindler wedge at temperature[1]

$$kT=a/2\pi\,. \tag{2.23}$$

The physical interpretation of this result may be stated as follows. Suppose an observer accelerates uniformly with acceleration a in Minkowski spacetime. Then to him, the structure of spacetime would appear to be "time independent" and thus he would view himself as "standing still," with the relevant "time translation" isometry given by b^a. If he were asked to formulate the quantum theory of a Klein-Gordon scalar field, he would naturally do so in terms of Eq. (2.8) for region I. What we have shown above is that when the field is in the state $|0_M\rangle$, our accelerating observer would describe it as being in the thermal state (2.22). We emphasize that Eq. (2.22) is *exactly* a thermal density matrix for Rindler states. Any particle detector used by the accelerating observer which measures the state of the field in terms of Rindler particles will determine that there is a thermal distribution of Rindler particles. However, this does *not* mean that a detector will respond exactly the same way as it would if placed in inertial motion in a (real) thermal bath of Minkowski particles. This is because the mode functions $\phi_{\mathrm{I}\omega}$ for Rindler particles are different from the mode functions for Minkowski particles. Another way of saying this is that the properties of a box of thermal radiation in inertial motion are measurably different from those of a box of thermal radiation in accelerating motion;[7] for example, the density distribution in the inertial box is uniform, whereas the density distribution in the accelerating box will vary with height because of the effective gravitational field. In the case of a scalar field, it turns out that a "monopole detector" cannot distinguish between the inertial and accelerating thermal distributions. However, in the case of an electromagnetic field, this difference can be seen.[8] We emphasize, however, that an accelerating observer still sees an *exactly* thermal distribution of particles; the only difference between the scalar and electromagnetic cases is that in the electromagnetic case, an isotropic, point detector is sensitive to the fact that the mode functions in the accelerating case are different from the mode functions in the inertial case.

Finally, we note that Eqs. (2.18) and (2.20) can be inverted to express $a_{R\mathrm{I}}(\phi_{\mathrm{I}\omega})$ in terms of Minkowski annihilation and creation operators. We obtain

$$a_{R\mathrm{I}}(\phi_{\mathrm{I}\omega})=[a_M(F_{1\omega})+e^{-\pi\omega/a}a^\dagger_M(F_{2\omega})]/(1-e^{-2\pi\omega/a})^{1/2}\,. \tag{2.24}$$

This formula will be used in Sec. IV.

III. MODEL PARTICLE DETECTOR

In this section we will introduce a simple model particle detector, designed to detect particles of the quantum field Φ. The model is essentially the "particle in a box" detector given previously[1] by one of us, but we shall simplify it further by assuming that only two energy levels are relevant and we also shall spell out more explicitly the coupling which we take between the detector and field.

We will restrict attention here to the case of a static spacetime with the detector "at rest," i.e., following the orbits of the static Killing field. However, our model will be applicable for any detector motion in any spacetime provided only that the region of spacetime occupied by the detector can be treated as approximately static. This will be the case if the detector is sufficiently small and the changes which it undergoes in curvature and acceleration are slow compared with the proper frequency of the particles it seeks to detect.

We shall take our detector to be a two-level system, with basis states denoted $|\uparrow\rangle$ and $|\downarrow\rangle$. We take the free Hamiltonian (with respect to the Killing time translations), H_D, of the detector to be given by

$$H_D=\Omega A^\dagger A \ . \tag{3.1}$$

Here $A^\dagger$ and A are the raising and lowering operators defined by

$$A\,|\downarrow\rangle=A^\dagger\,|\uparrow\rangle=0\ , \tag{3.2a}$$

$$A^\dagger\,|\downarrow\rangle=|\uparrow\rangle\ , \tag{3.2b}$$

$$A\,|\uparrow\rangle=|\downarrow\rangle\ . \tag{3.2c}$$

Thus, $|\downarrow\rangle$ is the ground state of H_D (with zero energy) and $|\uparrow\rangle$ is the excited state with energy Ω. It is convenient to introduce the operator C by

$$C=[A^\dagger,A]\ . \tag{3.3}$$

Then $\frac{1}{2}(I+C)$ is the projection operator onto $|\uparrow\rangle$ and $\frac{1}{2}(I-C)$ is the projection operator onto $|\downarrow\rangle$.

The coupling of the detector to the Φ field is assumed to be given by the interaction Hamiltonian

$$H_I=\epsilon(t)\int_\Sigma \Phi(x)[\psi(\underline{x})A+\psi^*(\underline{x})A^\dagger]\sqrt{-g}\,d^3\underline{x}\ . \tag{3.4}$$

Here $\epsilon(t)$ is the coupling constant, with explicit time dependence introduced to enable us to "turn on" and "turn off" the detector, $\psi(\underline{x})$ is a smooth function which vanishes outside the detector, and the integral is taken over the static slice Σ at time t. (Here we use x to denote a spacetime point and $\underline{x}$ to denote a point on Σ.) Thus, the total Hamiltonian of the field-detector system is

$$H=H_F+H_D+H_I\ , \tag{3.5}$$

where H_F is the free Klein-Gordon Hamiltonian of the Φ field.

Before continuing, we will relate this detector to that described previously by one of us.[1] There, the detector was taken to be a free particle in a rigid box. It was implicitly assumed that the coupling of the particle to the Φ field was of the form

$$H_I=\epsilon\int_\Sigma \Phi(x)\delta^{(3)}(\underline{x}-Q)\sqrt{-g}\,d^3\underline{x}\ , \tag{3.6}$$

where Q is the position operator for the particle in the box. (The particle was assumed to be nonrelativistic, so that Q was defined.) We expand $\delta^{(3)}(\underline{x}-Q)$ in terms of the energy eigenstates $|E_i\rangle$ as

$$\begin{aligned}\delta^{(3)}(\underline{x}-Q)&=\sum_{i,j}\langle E_i|\delta^{(3)}(\underline{x}-Q)|E_j\rangle|E_i\rangle\langle E_j|\\&=\sum_{i,j}\left[\int\delta^{(3)}(\underline{x}-q)\psi_i^*(q)\psi_j(q)\sqrt{-g}\,d^3q\right]|E_i\rangle\langle E_j|\\&=\sum_{i,j}\psi_i^*(\underline{x})\psi_j(\underline{x})|E_i\rangle\langle E_j|\ ,\end{aligned} \tag{3.7}$$

where ψ_i is the position-space wave function associated with $|E_i\rangle$. Thus, we obtain

$$H_I=\epsilon\sum_{i,j}\left[\int\Phi(x)\psi_i^*(\underline{x})\psi_j(\underline{x})\sqrt{-g}\,d^3\underline{x}\right]|E_i\rangle\langle E_j|\ . \tag{3.8}$$

In our case, we further simplify this model by restricting attention only to the $i=0,1$ subspace and dropping the diagonal terms $|E_0\rangle\langle E_0|$ and $|E_1\rangle\langle E_1|$. We obtain our model by writing $A=|E_0\rangle\langle E_1|$ and $\psi(\underline{x})=\psi_0^*(\underline{x})\psi_1(\underline{x})$ and making the coupling constant $\epsilon(t)$ explicitly time dependent.

In the following we will use both the interaction and Heisenberg representations to examine the evolution of the detector and the field. Although these representations are of course equivalent, the imagery produced by calculations done in these representations can be different and some questions are more easily answered in one than in the other.

In the Heisenberg representation, the states do not evolve while the operators do. The equations of motion for an observable O are

$$\frac{dO}{dt}=i[H,O]\ . \tag{3.9}$$

For the Hamiltonian of Eq. (3.5) this leads to the following equations of motion for Φ, A, $A^\dagger$, and C:

$$(\Box-m^2)\Phi=\epsilon(t)\int[\psi(\underline{x})A+\psi^*(\underline{x})A^\dagger]\sqrt{-g}\,d^3\underline{x}\ , \tag{3.10}$$

$$\frac{dA}{dt}=-i\Omega A+iC\epsilon(t)\int\Phi(t,\underline{x})\psi^*(\underline{x})\sqrt{-g}\,d^3\underline{x}\ , \tag{3.11}$$

$$\frac{dC}{dt}=2i\epsilon(t)\int[\psi(\underline{x})A-\psi^*(\underline{x})A^\dagger]\Phi(t,\underline{x})\sqrt{-g}\,d^3\underline{x}\ . \tag{3.12}$$

To first order in ϵ, the solutions to these equations are

$$\Phi(x)=\Phi_0(x)+\int G_R(x;t',\underline{x}')\epsilon(t')[\mathscr{A}_0 e^{-i\Omega t'}\psi(\underline{x}')+\mathscr{A}_0^{\dagger}e^{i\Omega t'}\psi^*(\underline{x}')]\sqrt{-g'}\,d^3\underline{x}'dt' , \tag{3.13}$$

$$A(t)=\mathscr{A}_0 e^{-i\Omega t}+C_0 e^{-i\Omega t}\int_{-\infty}^{t} e^{i\Omega t'}\epsilon(t')\int \Phi_0(t',\underline{x}')\psi^*(\underline{x}')\sqrt{-g'}\,d^3\underline{x}'dt' , \tag{3.14}$$

$$C(t)=C_0+2i\int_{-\infty}^{t}\epsilon(t')\int [\mathscr{A}_0 e^{-i\Omega t'}\psi(\underline{x}')-\mathscr{A}_0^{\dagger}e^{-i\Omega t'}\psi^*(\underline{x}')]\sqrt{-g'}\,d^3\underline{x}'dt' . \tag{3.15}$$

Here G_R is the retarded Green's function, Φ_0, A_0, and C_0 are the solutions to the free ($\epsilon=0$) equations which are equal to Φ, A, and C at times before the detector is turned on, and we have defined $\mathscr{A}_0=A_0e^{i\Omega t}$. Note that $\mathscr{A}_0$ and C_0 are constants.

In the interaction picture, on the other hand, the operators obey the free ($\epsilon=0$) field equations at *all* times. Thus if we choose the time at which the Heisenberg and interaction pictures agree to be prior to when the detector is switched on, the interaction picture observables will be just Φ_0, A_0, and C_0. In the interaction picture, a state $|s\rangle$ evolves via

$$|s,t\rangle=T\exp\left\{-i\int_{-\infty}^{t}\epsilon(t')\int \Phi_0(t',\underline{x}')[\psi(\underline{x}')A_0(t')+\psi^*(\underline{x}')A_0^{\dagger}(t')]\sqrt{-g'}\,d^3\underline{x}'dt'\right\}|s,-\infty\rangle , \tag{3.16}$$

where T denotes time ordering. To first order in ϵ we have

$$|s,t\rangle=|s,-\infty\rangle-i\left[\int_{-\infty}^{t}\epsilon(t')\int \Phi_0(t',\underline{x}')[\mathscr{A}_0 e^{-i\Omega t'}\psi(\underline{x}')+\mathscr{A}_0^{\dagger}e^{i\Omega t'}\psi^*(\underline{x}')]\sqrt{-g'}\,d^3\underline{x}'dt'\right]|s,-\infty\rangle . \tag{3.17}$$

We will be interested in the case in which the detector is in its lowest state $|\downarrow\rangle$ initially while the field initially is in a state containing $|n\rangle$ particles in the mode χ. Thus, the initial state $|s,-\infty\rangle$ is $|n;\downarrow\rangle$. We shall assume that $\epsilon(t)$ vanishes for $|t|>T\gg\Omega^{-1}$, and that it is roughly constant for $|t|<T$. Hence, the Fourier transform $\hat{\epsilon}(\omega)$ will be concentrated near $\omega=0$ with width much less than Ω. Using Eqs. (3.17) and (3.2) we find that the state of the system at times after the detector is switched off ($\epsilon=0$) is

$$\begin{aligned}|s,t>T\rangle&\equiv|s_\infty\rangle\\ &=|n;\downarrow\rangle-i\int e^{i\Omega t'}\epsilon(t')\psi^*(\underline{x}')\Phi_0(t',\underline{x}')\sqrt{-g'}\,dt'd^3\underline{x}'\,|n;\uparrow\rangle .\end{aligned} \tag{3.18}$$

Now, we use the fact that for any test function f we have

$$\Phi_0(f)\equiv\int f\Phi_0\sqrt{-g}\,d^4x=a(\Gamma_-^*)-a^{\dagger}(\Gamma_+) . \tag{3.19}$$

Here a^+ and a are the creation and annihilation operators for the free field Φ_0 and Γ_+ and Γ_- are the positive- and negative-frequency parts (with respect to the Killing time) of the retarded minus advanced solution Γ with source f,

$$\Gamma(x)=-2i\int [G_R(x;x')-G_A(x;x')]f(x')\sqrt{-g'}\,d^4x' . \tag{3.20}$$

This is most easily seen by writing

$$g(x)=-2i\int G_R(x;x')f(x')\sqrt{-g'}\,d^4x' . \tag{3.21}$$

Then we have $(\Box-m^2)g=-2if$ and

$$\begin{aligned}\int f\Phi_0\sqrt{-g}\,d^4x&=\frac{i}{2}\int(\Box-m^2)g\Phi_0\sqrt{-g}\,d^4x\\ &=\frac{i}{2}\int_\Sigma[g\nabla_\mu\Phi_0-(\nabla_\mu g)\Phi_0]dS^\mu ,\end{aligned} \tag{3.22}$$

where Σ is a Cauchy surface to the future of the support of f. On this surface, g agrees with Γ, and, as Γ is a solution of the free equation, we have

$$\int f\Phi_0\sqrt{-g}\,d^4x=\langle\Gamma^*,\Phi_0\rangle . \tag{3.23}$$

By splitting Γ into positive- and negative-frequency parts and using (2.7) we obtain (3.19).

We apply Eq. (3.19) to our case where $f=\epsilon(t)e^{i\Omega t}\psi^*(\underline{x})$. Since $\epsilon(t)$ is essentially constant f will have negligible positive-frequency part, so $\Gamma_+\approx 0$. Thus writing $\gamma=\Gamma_-^*$ we obtain

$$|s_\infty\rangle=|n;\downarrow\rangle-i|\uparrow\rangle\otimes a(\gamma)|n\rangle . \tag{3.24}$$

However we have $a(\gamma)|n\rangle=\sqrt{n}\,\langle\gamma,\chi\rangle\,|n-1\rangle$ where $|n-1\rangle$ denotes the state with $n-1$ particles in the mode χ. Thus we find

$$|s_\infty\rangle=|n;\downarrow\rangle-i\sqrt{n}\,\langle\gamma,\chi\rangle\,|n-1;\uparrow\rangle . \tag{3.25}$$

In particular, the lowest-order probability of finding the detector is the state $|\uparrow\rangle$ is

$$\begin{aligned}P_\uparrow&=n\,|\langle\gamma,\chi\rangle|^2\\ &=n\left|\int\epsilon(t)\,e^{i\Omega t}\psi^*(\underline{x})\chi(t,\underline{x})\sqrt{-g}\,d^4x\right|^2 .\end{aligned} \tag{3.26}$$

Thus, the excitation probability is just equal to the number of quanta present times a cross-section factor which represents the overlap between the mode of interest and the detector. Furthermore, according to Eq. (3.25), to lowest order in ϵ the detector will be excited if and only if one quantum of the field has been absorbed. Thus, our simple model does function properly as a particle detector.

We now examine the behavior of the detector-field system when the field is initially in a thermal state. Since a thermal bath consists of an incoherent superposition of n particles in each mode of energy E—with probability proportional to the Boltzmann factor e^{-nE} [see Eq. (2.22) above]—the behavior of the detector in a thermal bath follows immediately from its behavior in n-particle eigenstates as calculated above. In particular, the excitation probability in this case will just be given by [using (3.26)]

$$P_\uparrow = \sum_{\chi_\omega, n} p(n,\chi_\omega) n \, |\langle \gamma, \chi_\omega\rangle|^2$$
$$= \sum_{\chi_\omega} \frac{e^{-\beta\omega}}{1-e^{-\beta\omega}} |\langle \gamma,\chi_\omega\rangle|^2$$
$$\approx \frac{e^{-\beta\Omega}}{1-e^{-\beta\Omega}} \langle \gamma,\gamma\rangle \,, \tag{3.27}$$

where $p(n,\chi_\omega)$ is the probability that the state has n quanta in the mode χ_ω, ω is the frequency with respect to the Killing time, and χ_ω are a complete set of positive-ω-frequency states in the vicinity of the detector. (Since γ is positive frequency and is by assumption a state whose frequency is very near $\omega=\Omega$, the last line is a reasonable approximation.)

Nevertheless, the result of the interaction of the detector with the thermal bath is, at first sight, rather surprising. If the detector is found in its excited state, we found above that a field quantum is absorbed and therefore one might expect that the expected energy of the Φ field will have decreased by the energy Ω that the detector has absorbed. However, this obvious expectation is wrong. We now shall show that when the detector is excited, the expected energy of the Φ field goes up.

Since $\frac{1}{2}(I+C_0)$ is the projection operator onto the $|\uparrow\rangle$ state, the expected stress energy in the Φ field in the case where the detector is excited is given by

$$\langle T_{\mu\nu}\rangle_\uparrow = \frac{\Sigma p(s) \langle s_\infty | \frac{1}{2}(I+C_0) T_{\mu\nu} \frac{1}{2}(I+C_0) | s_\infty\rangle}{\Sigma p(s) P_\uparrow(s)} \,, \tag{3.28}$$

where $T_{\mu\nu}$ is the stress-energy operator of the Φ field, $p(s)$ is the probability that the initial state was $|s\rangle$, and $P_\uparrow(s)$ is the detector excitation probability when the initial state was $|s\rangle$. In the case of an initial thermal bath, a direct calculation yields

$$\delta\langle T_{\mu\nu}\rangle \equiv \langle T_{\mu\nu}\rangle_\uparrow - \langle T_{\mu\nu}\rangle_0$$
$$= \frac{e^{-\beta\Omega}}{1-e^{-\beta\Omega}} \frac{t_{\mu\nu}(\gamma)}{\langle\gamma,\gamma\rangle} \,, \tag{3.29}$$

where $\langle T_{\mu\nu}\rangle_0$ denotes the expected stress-energy of a thermal bath and $t_{\mu\nu}(\gamma)$ is the classical stress-energy of the mode γ,

$$t_{\mu\nu}(\gamma) = \tfrac{1}{2}[\nabla_\mu\gamma^* \nabla_\nu\gamma - \tfrac{1}{2} g_{\mu\nu}(\nabla_\alpha\gamma^*\nabla^\alpha\gamma + m^2|\gamma|^2)] + \text{c.c.} \tag{3.30}$$

Since t_{00} is positive, this shows that the expected field energy *increases* upon detection.

The reason for this behavior can be understood as follows. The initial state of the quantum field is not an eigenstate of energy. The act of detection not only removes a particle of energy Ω from the field but it also performs a (partial) measurement of the state of the field, since the detector is most likely to be excited if a large number of particles were initially present, as can be seen from the factor of n appearing in Eq. (3.26). Thus, the fact that the detector became excited weights the high-particle-number states more heavily in the distribution and indicates that a larger number of particles that originally expected were present initially. As a dramatic example of this effect, suppose that the initial state of the field were chosen to be $|0\rangle + (1/\sqrt{n})|n\rangle$ with n large. Then the initial expected energy is $\approx E$, where E is the energy of a single quantum. However, if detection occurs, then the expected field energy becomes $(n-1)E \gg E$. In the case of a thermal bath the original probability of having n particles present in the mode γ of energy Ω is proportional to $e^{-n\beta\Omega}$. Because of the factor of $\sqrt{n}$ in Eq. (3.25), if a particle is detected, the probability of having n particles present in that mode becomes proportional to $(n+1)e^{-n\beta\Omega}$. Thus, for a thermal distribution, the "partial measurement" effect dominates the energy absorption.

At first sight, it might appear that the above mechanism provides us with a way of extracting an infinite amount of energy from a (finite) thermal bath, i.e., an ordinary box filled with a thermal distribution of quanta (so that the initial expected field energy is finite). Namely, we insert our detector into the bath and bring it out. If the detector is unexcited, then according to Eq. (3.25) no change has occurred in the state of the field. But if the detector is excited, we have gained energy Ω in the detector *and* increased the expected field energy in the box. We then can remove the detector and de-excite it (thereby extracting energy Ω) and repeat the process indefinitely. (Although the distribution of field quanta no longer is thermal, the expected field energy continues to increase each time detection occurs.) Thus, it appears that, with unit probability, we can extract an arbitrarily large amount of field energy from a system whose initial expected energy is finite.

The key to the resolution of this apparent paradox arises from the fact that at each stage, the increase in the expected field energy after detection is second order in the coupling constant ϵ. Thus, to be consistent, we must include all contributions to $|s_\infty\rangle$ of order ϵ^2. To order ϵ^2, the detector may absorb a quantum, thereby jumping to its excited state, and then emit a quantum thereby de-exciting and leaving the field in the same state. The probability that this occurs can be calculated directly from Eq. (3.16), but is obtained much more simply from conservation of probability: If n particles in mode γ are present we have, to second order in ϵ, $P_\uparrow = n\langle\gamma,\gamma\rangle$, so to second order the probability that the system ends up in the original state $|n;\downarrow\rangle$ must be $(1-n\langle\gamma,\gamma\rangle)$. This means that for a detector placed in a thermal bath, if *no* quanta are detected, then the probability distribution for having n particles in mode γ becomes proportional to $e^{-n\beta\Omega}(1-n\langle\gamma,\gamma\rangle)$. Thus, the "partial measurement" effect causes the expected field energy to *decrease* when no detection occurs in just the right way to compensate for the increase in expected field energy when detection occurs, thereby yielding conservation of expected energy for the total system. If we try to extract energy by repeatedly inserting our detector into a thermal bath, we are as likely to have the net effect of decreasing the expected energy of the total system by frequently finding the detector unexcited as we are to increase the total expected energy by occasionally finding the detector to be excited.

IV. RESPONSE OF THE FIELD TO AN ACCELERATING PARTICLE DETECTOR

We are now ready to analyze the response of the field to an accelerating particle detector in flat spacetime. The response of the detector to the field when the field is in its Minkowski vacuum state has been analyzed often before. The result, that the detector responds as though immersed in a thermal bath can be derived from our results in Secs. II and III. A uniformly accelerating detector can be taken to follow an orbit of one of the boost Killing fields. In Sec. II we showed that if the initial state of the field was the Minkowski vacuum $|0_M\rangle$, then that initial state expressed in terms of Rindler particles is a thermal state with temperature $a/2\pi$. In Sec. III, Eq. (3.27), we calculated the response of a model detector to a thermal bath (with respect to the particles defined in terms of the Killing field whose orbit the detector follows). We can apply the results directly to the accelerated detector by setting $\beta=2\pi/a$. Our aim in this section is to provide a description of the effect the detector has on the field, especially when the detector is found to be excited, after the interaction is switched off. In addition we will raise and resolve a number of apparent paradoxes.

First, we shall use the interaction representation and express the final state of the detector-field system in terms of Minkowski particle states. As in Sec. II, we shall use t to denote Minkowski time and τ to denote Rindler time. We choose the initial state of the field to be $|0_M\rangle$ and the initial state of the detector to be $|\downarrow\rangle$. Then the same derivation as led to Eq. (3.24) yields that the state of the system at late times (after the detector is turned off) is

$$|s_\infty\rangle=|0_M;\downarrow\rangle-ia_{R\mathrm{I}}(\gamma)\,|0_M;\uparrow\rangle\ , \tag{4.1}$$

where γ is the complex conjugate of the negative-frequency part of the Rindler mode Γ defined by Eq. (3.20) with $f=\epsilon(\tau)e^{i\Omega\tau}\psi^*$. Thus, γ is peaked sharply about Rindler frequency $\omega=\Omega$. But, at the end of Sec. II we derived an expression for the Rindler annihilation operator of such a mode in terms of Minkowski annihilation and creation operators. Thus, using Eq. (2.24), we obtain the following expression for the late-Minkowski-time state:

$$\begin{aligned}|s_\infty\rangle&=|0_M;\downarrow\rangle-i(1-e^{-2\pi\Omega/a})^{-1/2}[a_M(F_{1\Omega})+e^{-\pi\Omega/a}a_M^\dagger(F_{2\Omega})]\,|0_M;\uparrow\rangle\\&=|0_M;\downarrow\rangle-i(1-e^{-2\pi\Omega/a})^{-1/2}e^{-\pi\Omega/a}a_M^\dagger(F_{2\Omega})\,|0_M;\uparrow\rangle\ ,\end{aligned} \tag{4.2}$$

where $F_{1\Omega}$ is given by Eq. (2.17) with $\phi_{\mathrm{I}\Omega}=\gamma$ and $F_{2\Omega}$ is defined by Eq. (2.19). Thus, $a_M^\dagger(F_{2\Omega})|0_M;\uparrow\rangle$ is the state when the detector is excited and one Minkowksi field quantum in mode $F_{2\Omega}$ is present. Note that both γ and $F_{2\Omega}$ are *not* normalized and, indeed, their amplitudes are proportional to the coupling constant ϵ.

Thus, Eq. (4.2) shows that in the inertial viewpoint, *the detection of a Rindler particle corresponds to emission of a Minkowski particle in mode* $F_{2\Omega}$. The inertial observer might well interpret the transition made by the detector as due to a radiation reaction effect produced by the emission of a particle by the "detector." Note that in this model, emission of a Minkowski particle occurs only when the detector makes a transition, so this transition cannot be interpreted as being caused by self-absorption of Minkowski quanta that the detector previously emitted by bremsstrahlung or other processes.[9]

We now may resolve the apparent paradox regarding energy which was raised in Sec. I. If the detector is excited, then the Φ field is in the state with one Minkowski particle in mode $F_{2\Omega}$ and its expected stress energy is $t_{\mu\nu}(F_{2\Omega})/\langle F_{2\Omega},F_{2\Omega}\rangle$ where $t_{\mu\nu}$ is the classical stress energy given by Eq. (3.30). Thus, the emission of a Minkowski particle occurring when the detector is excited increases the Φ field energy everywhere.[10] In particular, using the definition of $F_{2\Omega}$ in terms of γ, we find that the expected change in $T_{\mu\nu}$ in Rindler region I is

$$\delta\langle T_{\mu\nu}\rangle_{\mathrm{I}}=\frac{e^{-2\pi\Omega/a}}{(1-e^{-2\pi\Omega/a})}\frac{t_{\mu\nu}(\gamma)}{\langle\gamma,\gamma\rangle}\ . \tag{4.3}$$

This, of course, agrees with Eq. (3.29) with $\beta=2\pi/a$. Note, however, how remarkably different are the inertial and accelerating interpretations of why the expected boost energy increases in Rindler wedge I. The inertial observer simply says that field energy increases because a particle was emitted. The accelerating observer says that the field energy was actually decreased by absorption of a particle but because a partial measurement was performed during the absorption process, the net effect was to increase the field energy.

Note that the Minkowski mode $F_{2\Omega}$ is, in fact, mostly located in Rindler wedge II [see Eqs. (2.19) and (2.17)]. Since wedges I and II of Fig. 1 are noncausally related, this might suggest that we can violate causality by using a detector in region I to emit the particle $F_{2\Omega}$, whose influence then can be felt in region II. Is it possible to send a signal from region I to region II in this manner?

The answer is, of course, no. This is most easily seen by going to the Heisenberg representation. In the Heisenberg representation, the field Φ is given by

$$\Phi(x)=\Phi_0(x)+\int G_R(x;\tau',\underline{x}')\epsilon(\tau')[A(\tau')\psi(\underline{x}')+A^\dagger(\tau')\psi^*(\underline{x}')]\sqrt{-g'}\,d\tau'd^3\underline{x}'\ . \tag{4.4}$$

The key point to notice is that the field Φ is changed only to the future of the region containing the switched on ($\epsilon\neq0$) detector because of the retarded Green's function G_R, in the expression for the field Φ. This means that the expectation value of *any* operator which depends only on the field Φ smeared with test functions with support in region II of the Rindler spacetime will be completely independent of the detector and will depend only on the free initial field Φ_0. In other words, if $O(\Phi)$ is an operator which depends only on $\Phi(x)$ with x in region II, then

$O(\Phi)=O(\Phi_0)$ and the expectation value of O will be unchanged by the interaction of the field with the detector. Since any observation of the field which is restricted to region II will be a measurement of such an operator, any such observation will give results indistinguishable from the vacuum. In particular if one places a particle detector into region II to try to detect the particle in mode $F_{2\Omega}$, one would either have to switch that detector on and off while it was in region II, giving rise to detector excitations from the switching process, or one would have to allow the detector to remain on in the future Rindler wedge (region III in Fig. 1), in which case one could ascribe the excitation of that detector to the causal influence of the accelerated detector in region I.

Nevertheless, correlations *do* exist between the measurements of the state of the detector and properties of the Φ field in region II. In particular, if the detector is in its excited state, the expected energy density of Φ in region II is positive. From where does this correlation between the detector and field in causally disconnected regions arise? The answer is again most easily seen in the Heisenberg representation. Given some operator $O(\Phi)$ which again depends only on the field Φ in region II, and some other operator $Q(A,A^\dagger)$ which depends on the dynamic variables of the detector, we find

$$\langle 0_M;\downarrow|\,Q(A,A^\dagger)O(\Phi)\,|0_M;\downarrow\rangle = \langle 0_M;\downarrow|\,Q(A,A^\dagger)O(\Phi_0)\,|0_M;\downarrow\rangle \;, \quad (4.5)$$

so that again any such expectation value does not depend on the changes induced in the field by the detector. However, from Eq. (3.14) we have to first order in ϵ

$$A=\mathscr{A}_0 e^{-i\Omega\tau}+C_0 e^{-i\Omega\tau}\int^{\tau} e^{i\Omega\tau'}\epsilon(\tau')\int \psi^*(\underline{x}')\Phi_0(x')\sqrt{-g'}\times d^3\underline{x}'d^3\tau' \;. \quad (4.6)$$

The expectation value therefore can be written as

$$\langle 0_M;\downarrow|\,Q(A,A^\dagger)O(\Phi)\,|0_M;\downarrow\rangle = \langle 0_M;\downarrow|\,\widetilde{Q}(A_0,A_0^\dagger,\Phi_0)O(\Phi_0)\,|0_M;\downarrow\rangle \;, \quad (4.7)$$

where the dependence of $\widetilde{Q}$ on Φ_0 is that due to $\Phi_0(x)$ with x in region I. However, in the vacuum state, the field Φ_0 is correlated even in causally disconnected regions of spacetime. For example, we have for a massless field

$$\langle 0|\,\Phi_0(x)\Phi_0(x')+\Phi_0(x')\Phi_0(x)\,|0\rangle \propto \frac{1}{\sigma} \;, \quad (4.8)$$

where σ is the square of the geodesic distance between x and x'. Thus, this free field vacuum expectation value is nonzero when x and x' are spacelike separated. The correlation between the state of the detector and the state of the field in causally disconnected regions of spacetime is a result of such initial correlations in the field Φ_0 in the vacuum state. However, as in the Einstein-Podolsky-Rosen effect, one cannot use these correlations to send signals. As shown above, any experiment which is sensitive only to the field Φ in region II will give results which are indistinguishable from those performed in the vacuum state. Only when the outcomes of such measurements are compared with the final state of the detector do interesting correlations arise. However, the existence of such correlations does not violate causality.

As noted above, in lowest order the excitation of the detector is associated with a positive expected field energy in region II. Therefore, failure of the detector to become excited must be correlated with negative expected field energy in region II, since the total expected field energy in region II must be that of the vacuum, i.e., zero. However, this appears to contradict our expression (4.2) for $|s_\infty\rangle$, which shows that if the detector state is $|\downarrow\rangle$, then the field state is $|0_M\rangle$ and hence has vanishing expected energy in region II. The resolution of this apparent contradiction is very similar to the "infinite-energy-extraction paradox" discussed at the end of the previous section. Again, the increase in expected energy in region II when the detector is excited is quadratic in ϵ, so for consistency we must keep all second-order contributions to $\langle T_{00}\rangle$. When we do so, we find a second-order term in $|s_\infty\rangle$ which is of the form $|\downarrow\rangle\otimes(a'^\dagger_M a''^\dagger_M|0_M\rangle)$ where $a'^\dagger_M$ and $a''^\dagger_M$ are creation operators for certain Minkowski modes of Φ. In other words, there is a nonvanishing amplitude for the detector to emit two Minkowski particles. The interference term in T_{00} between this second-order term and the zeroth-order term $|0_M;\downarrow\rangle$ gives rise to a negative expected field energy in region II which exactly cancels the positive contribution from the first-order term in $|s_\infty\rangle$. This cancellation will occur to all orders in ϵ, a fact which is obvious in the Heisenberg representation but not as easily demonstrated in the interaction representation.

Thus, we have succeeded in providing an inertial interpretation of what happens to an accelerating particle detector in Minkowski spacetime and we have resolved some apparent paradoxes regarding energy conservation and causality. The inertial description of the process which occurs when the detector becomes excited is simply that it radiates a field quantum. But our analysis suggests a rather surprising viewpoint on this radiation process: it seems as though the detector is excited by swallowing part of the vacuum fluctuation of the field in the region of spacetime containing the detector. This liberates the correlated fluctuation in a noncausally related region of the spacetime to become a real particle.

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[10]A more detailed calculation of the form of the stress-energy tensor from an accelerated detector will be given in the thesis of J. Cant (unpublished).