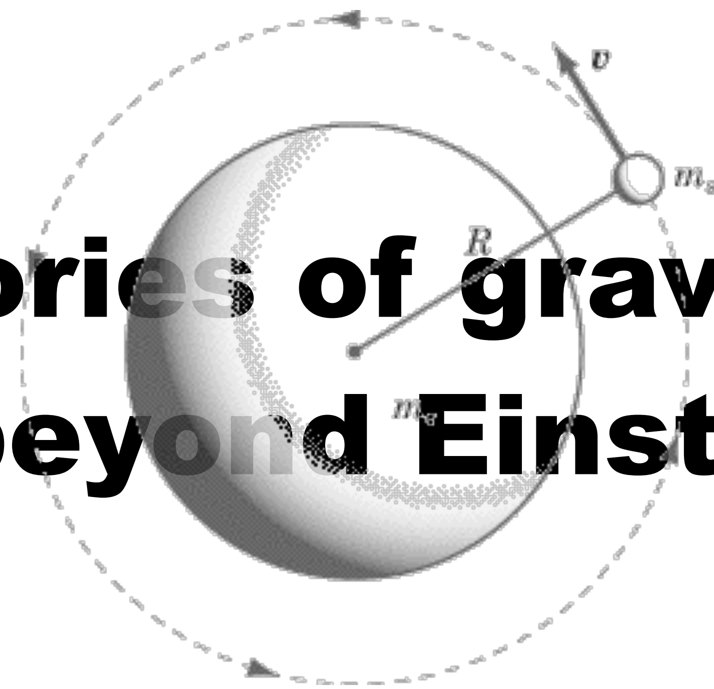
Docentföreläsning



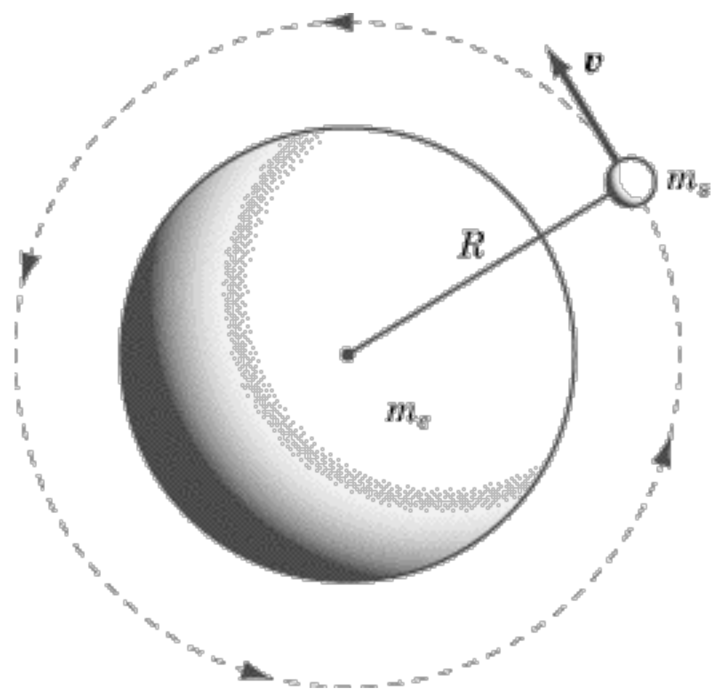
Theories of gravitation beyond Einstein

Stockholm University

Marcus Berg
CoPS
OKC

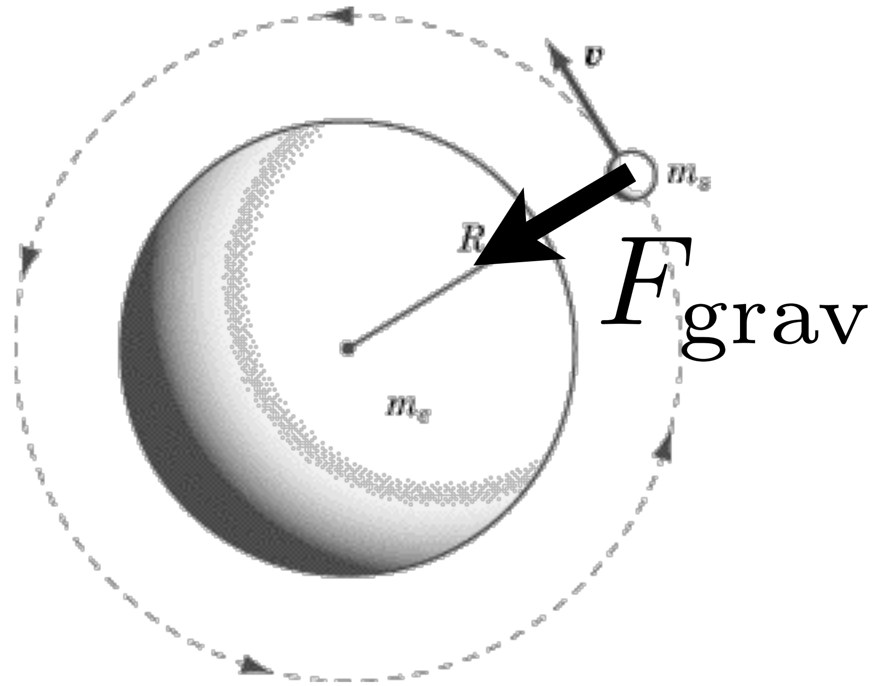
Oskar Klein centre

Newtonian gravity (1686)



Newtonian gravity (1686)

$$F_{\mathrm{grav}} = G_N \frac{m_e m_s}{R^2}$$



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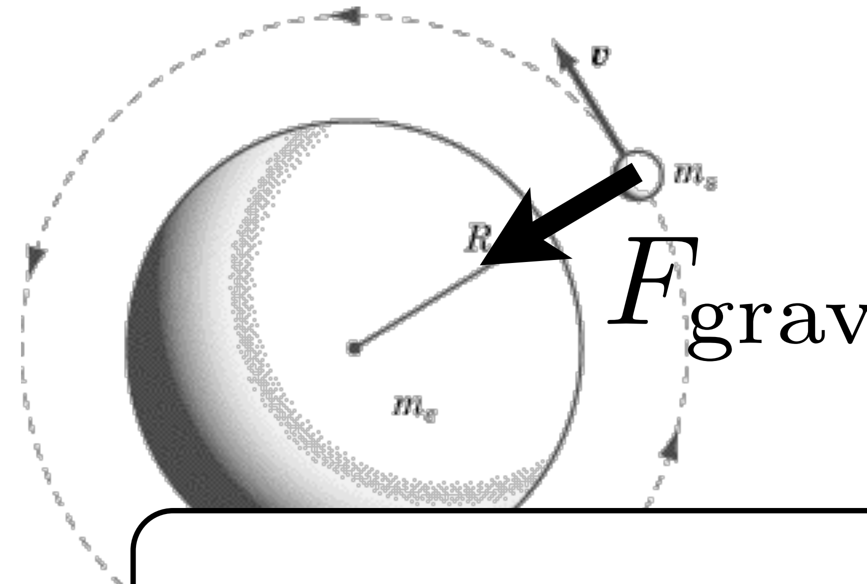
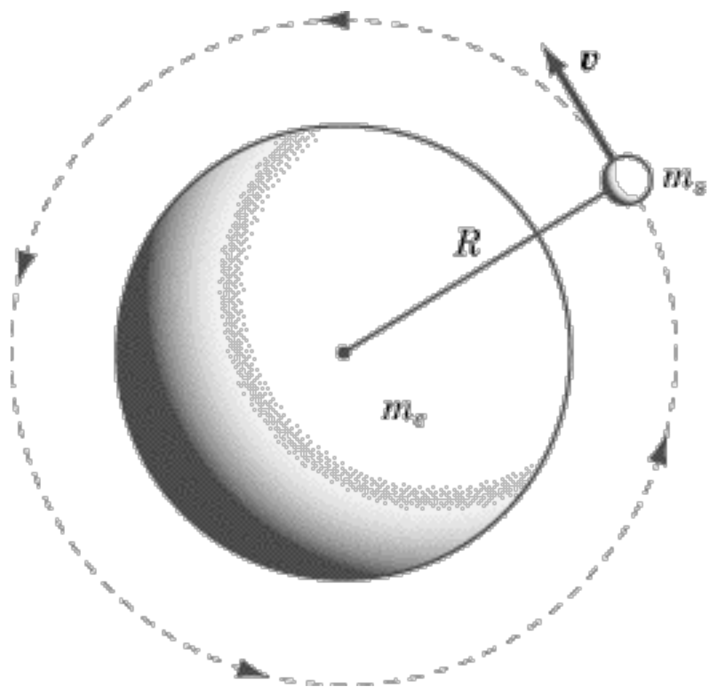


$$G_N \frac{m_e m_s}{R^2} = m_s \frac{v^2}{R}$$

$$v = \sqrt{\frac{G_N m_e}{R}}$$

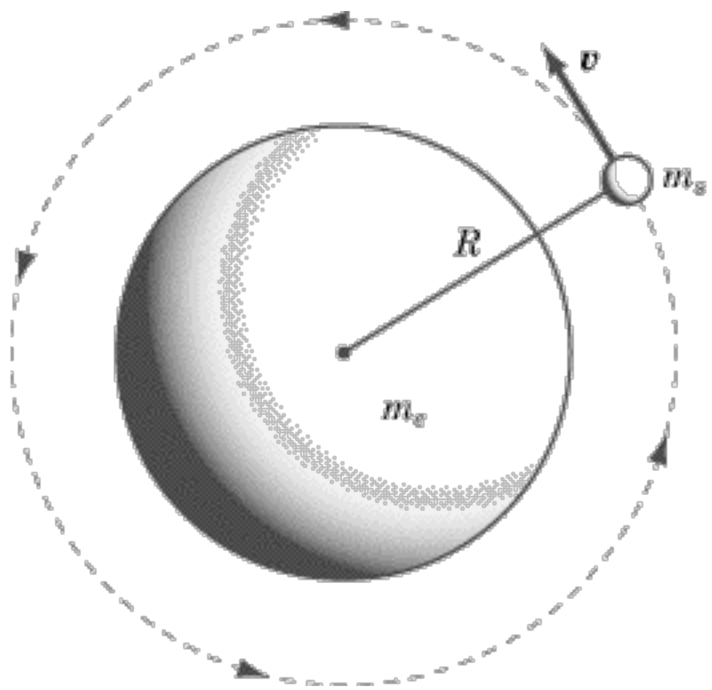
Newtonian gravity (1686)



the gravitational *potential* is a "scalar field" from which the force F_{grav} is derived

Maria Pietilä-Holmner
(Umeå, Sweden)
Gold medal, World Cup
Aspen Nov 28 2010

Newtonian gravity (1686)



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$$\vec{F} = -m\vec{\nabla}\phi$$

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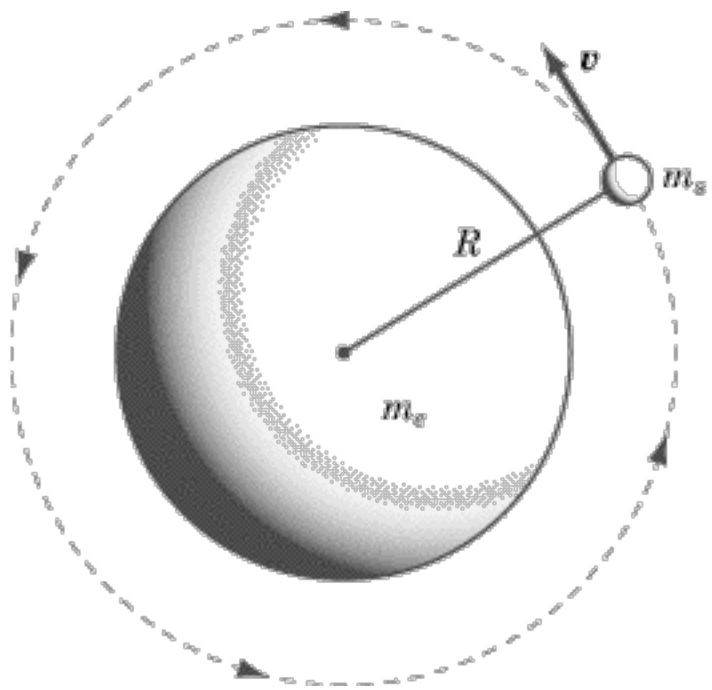
Bräntbacken, Umeå

Newtonian gravity (1686)



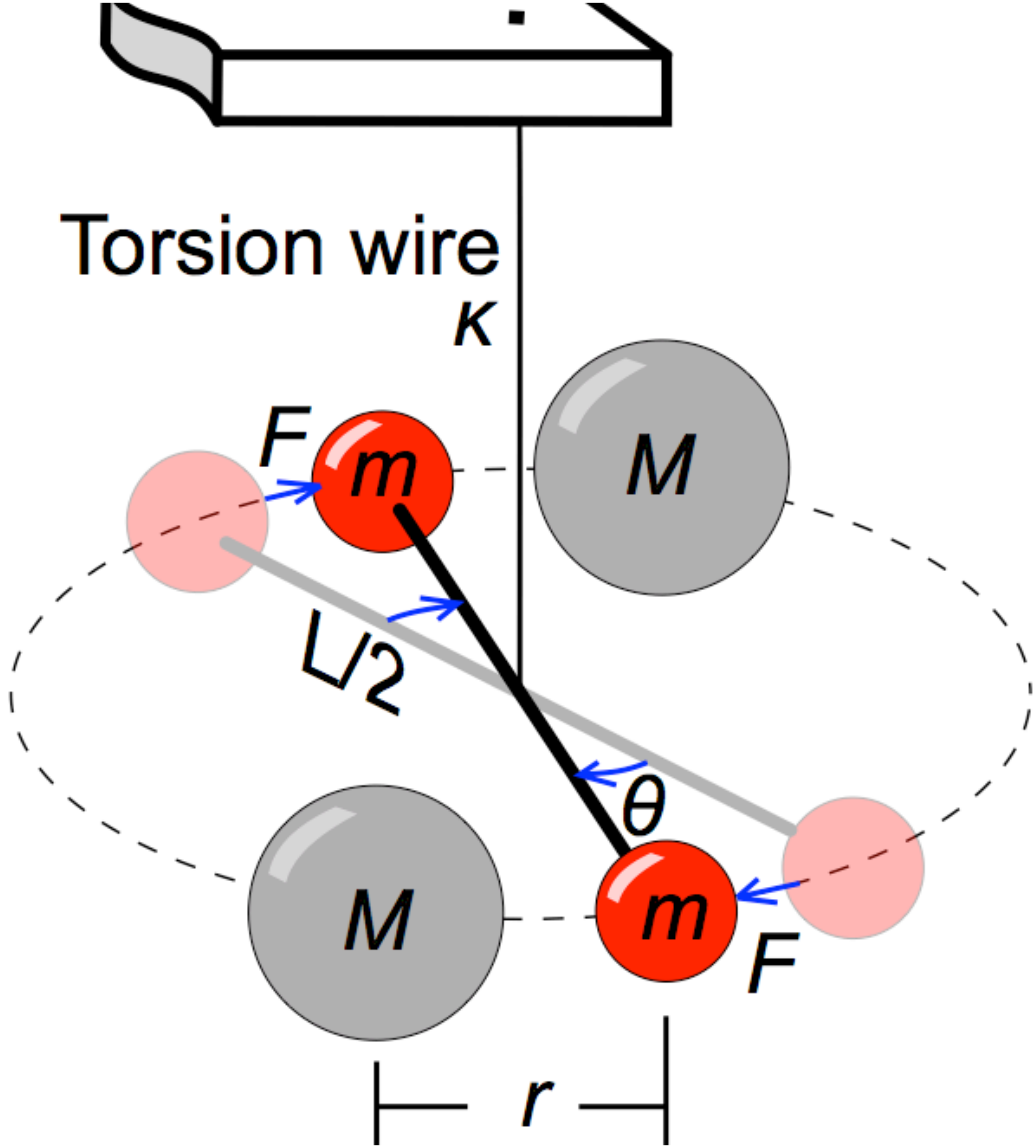
the gravitational *potential* is a "scalar field" from which the force is derived:

$$\vec{F} = -m\vec{\nabla}\phi$$

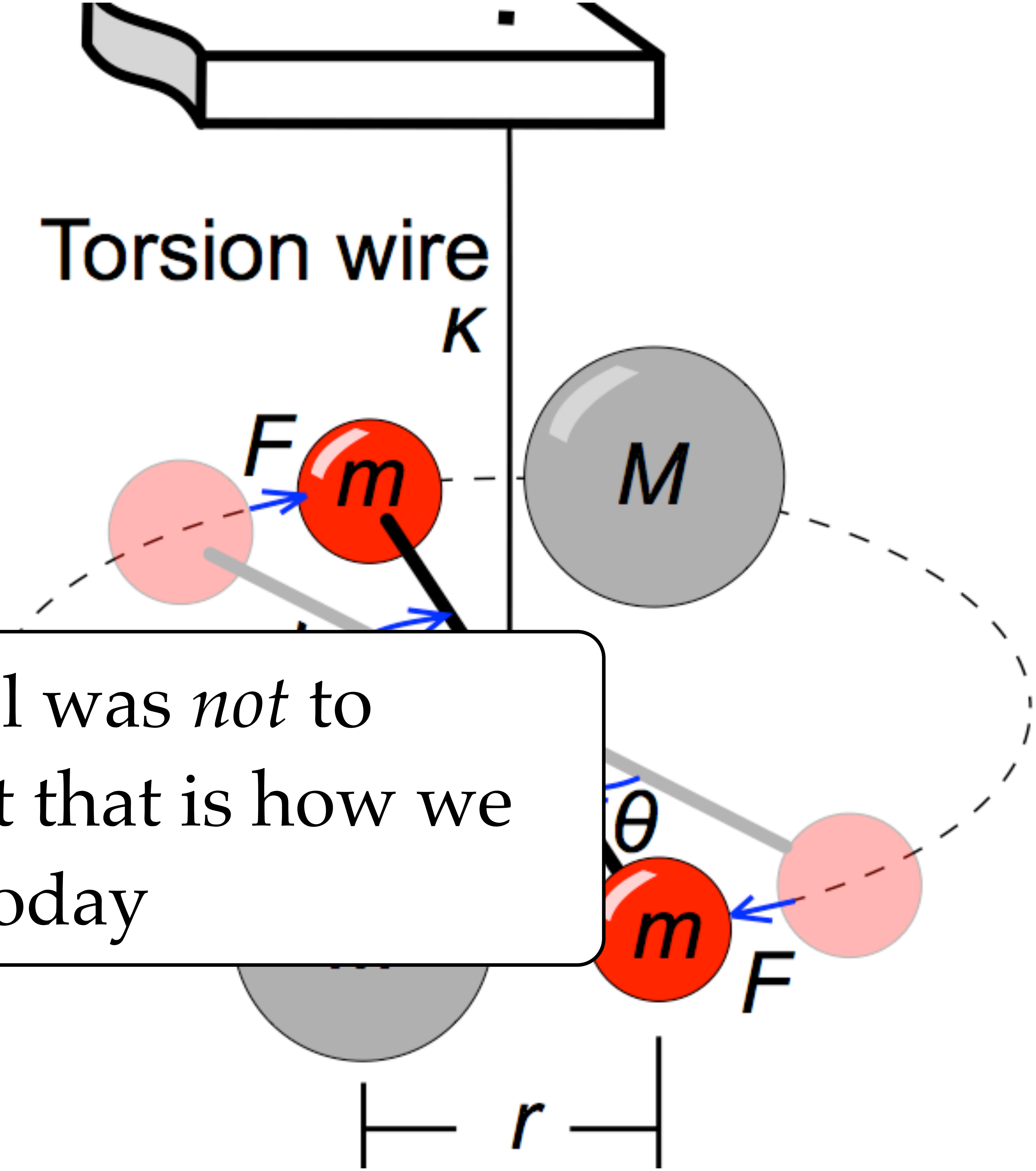
The force law becomes: $\nabla^2\phi = 4\pi G_N \cdot \rho$

where ρ is the mass *density*.

Cavendish experiment (1797)



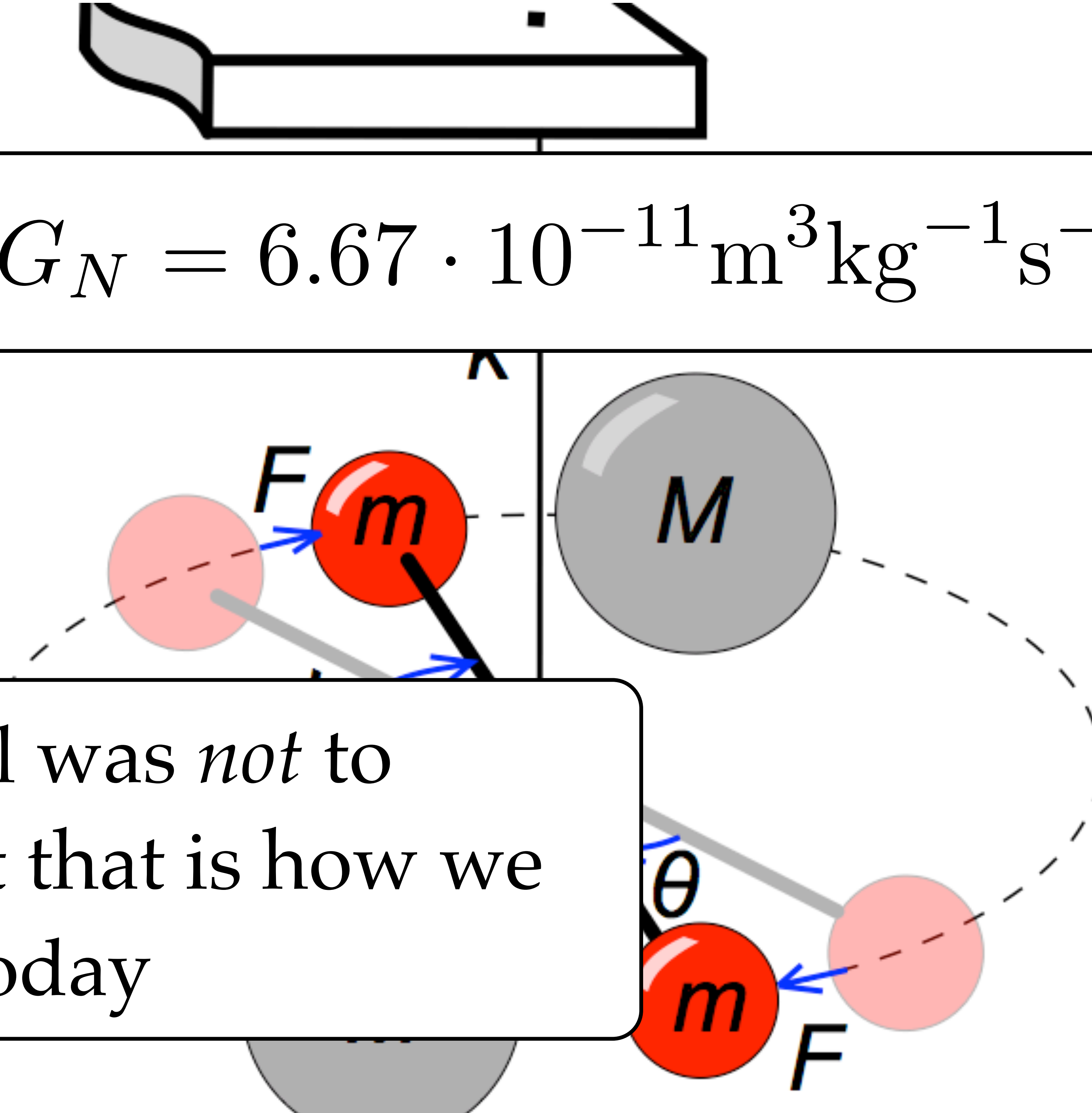
Cavendish experiment (1797)



Cavendish's goal was *not* to measure G_N, but that is how we can interpret it today

Cavendish experiment (1797)

$$G_N = 6.67 \cdot 10^{-11} \mathrm{m}^3 \mathrm{kg}^{-1} \mathrm{s}^{-2}$$

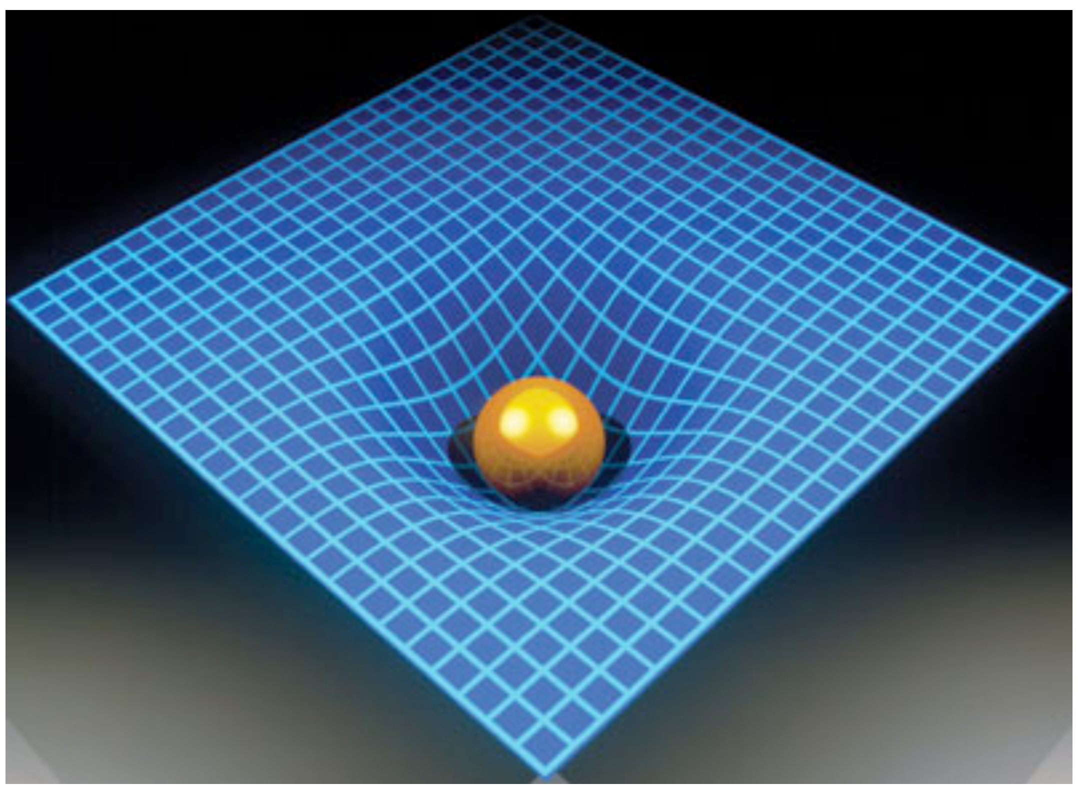


Cavendish's goal was *not* to measure G_N, but that is how we can interpret it today

$$G_N^{\mathrm{year}\,1797} = 6.74 \cdot 10^{-11} \mathrm{m}^3 \mathrm{kg}^{-1} \mathrm{s}^{-2}$$

Einstein's theory of gravitation (1915)

(General relativity)



Einstein's theory of gravitation (1915)

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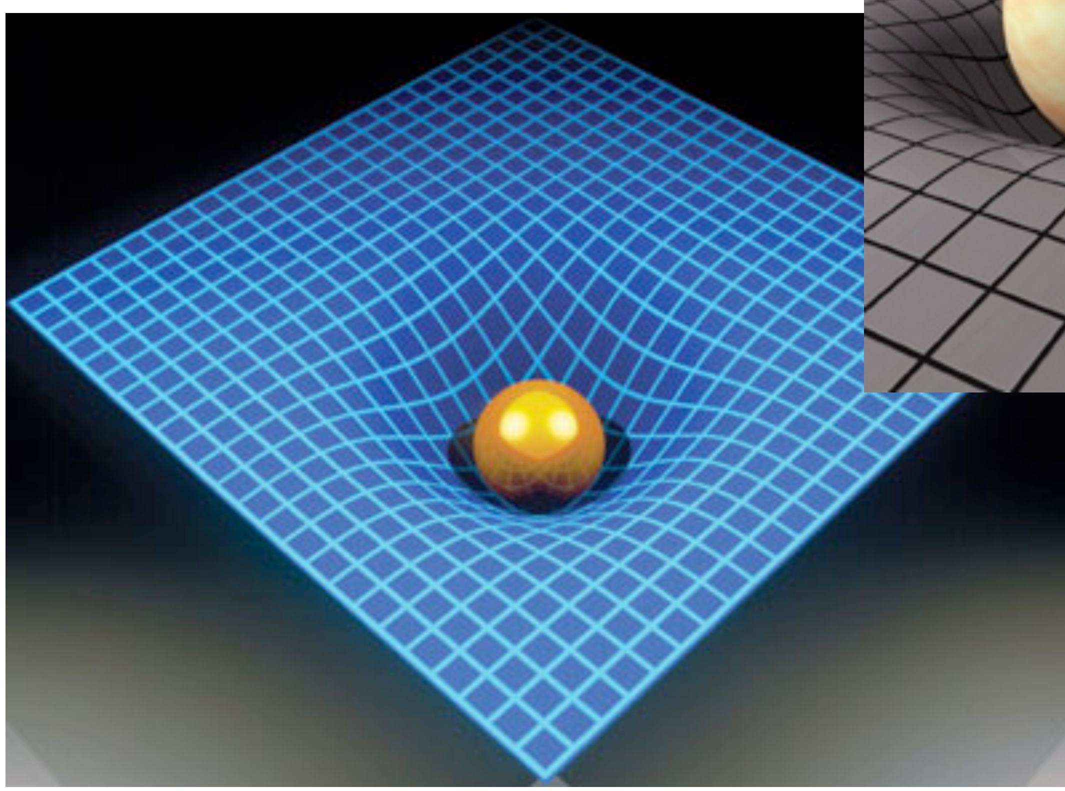

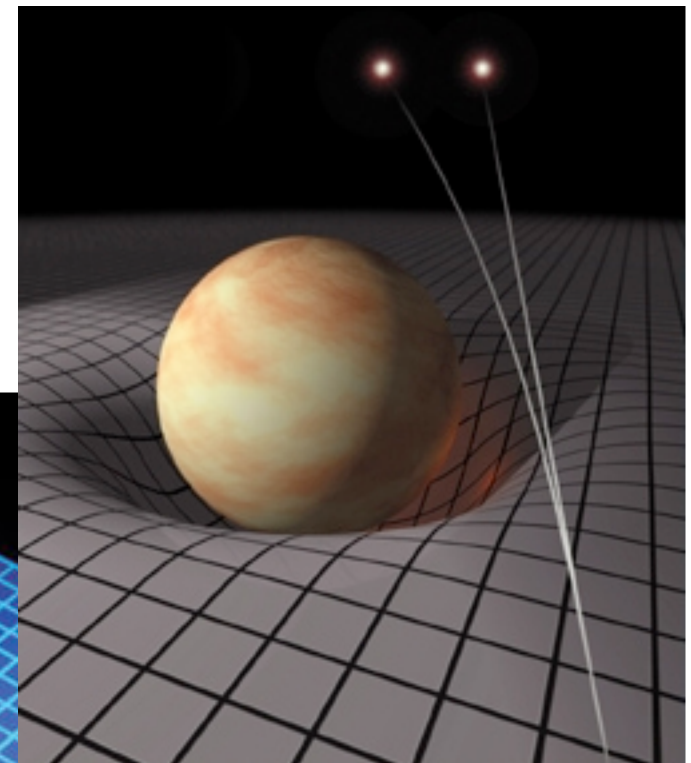

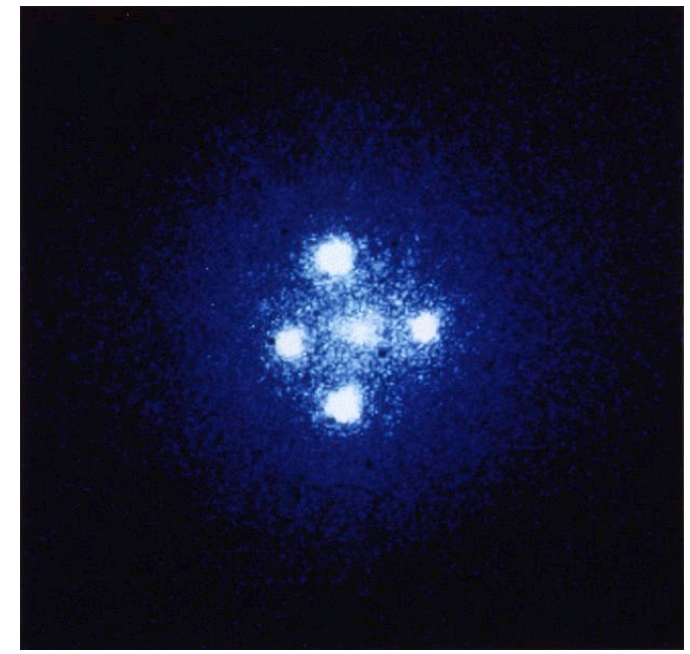

Einstein's theory of gravitation (1915)

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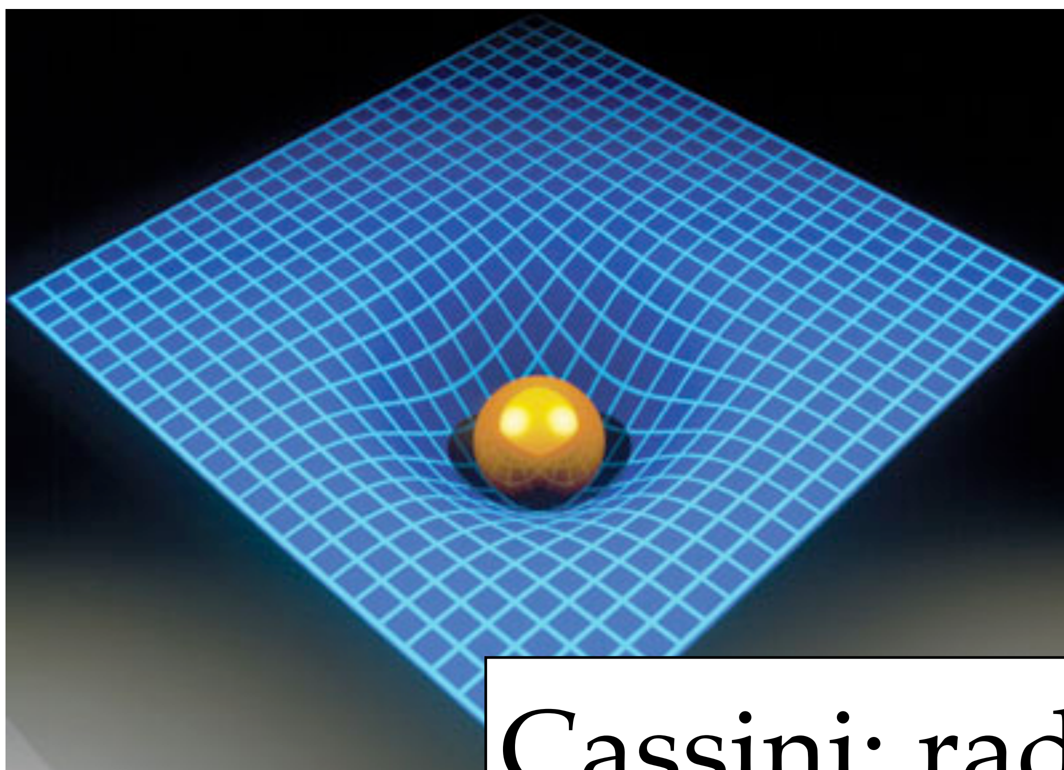
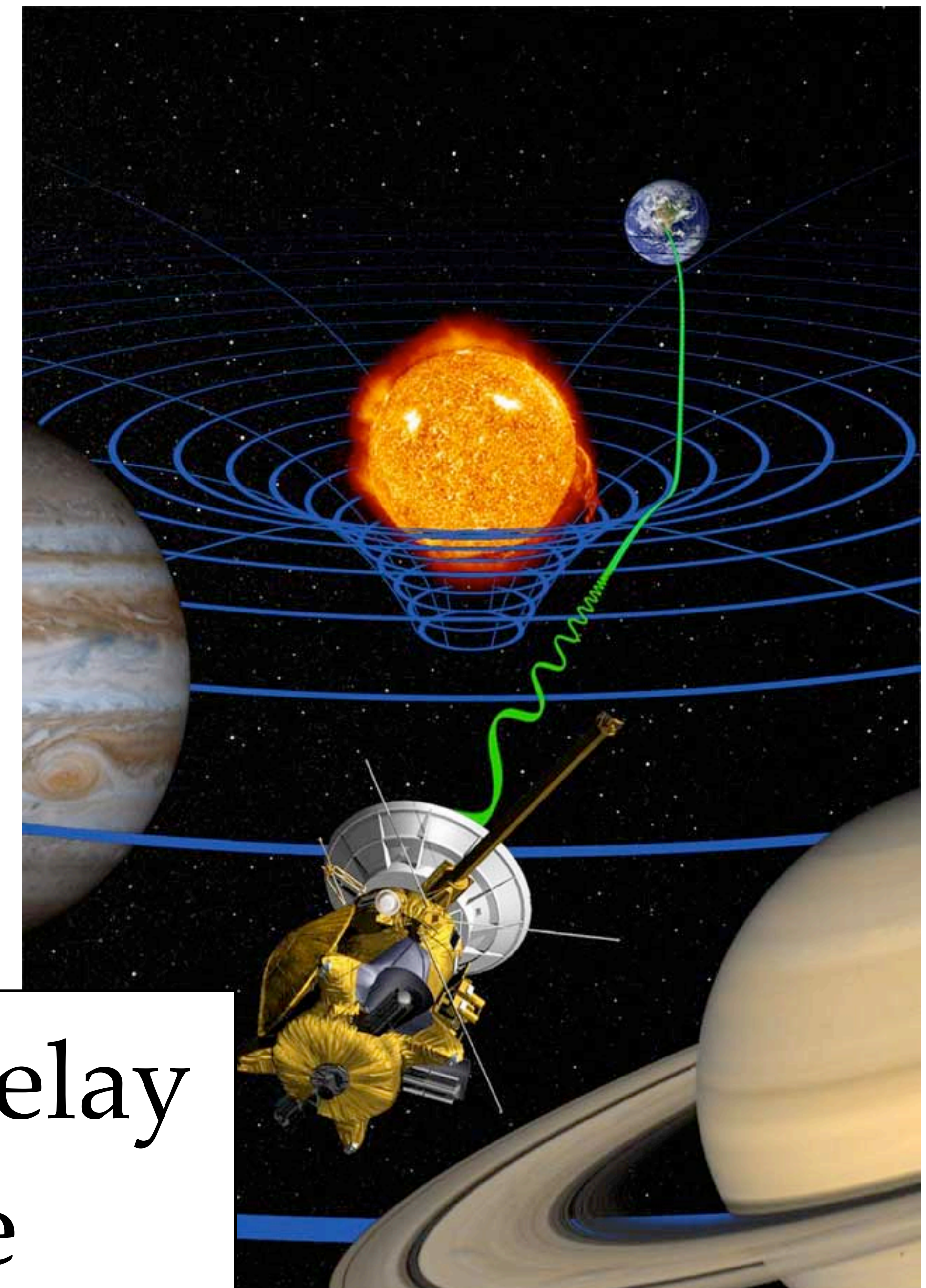
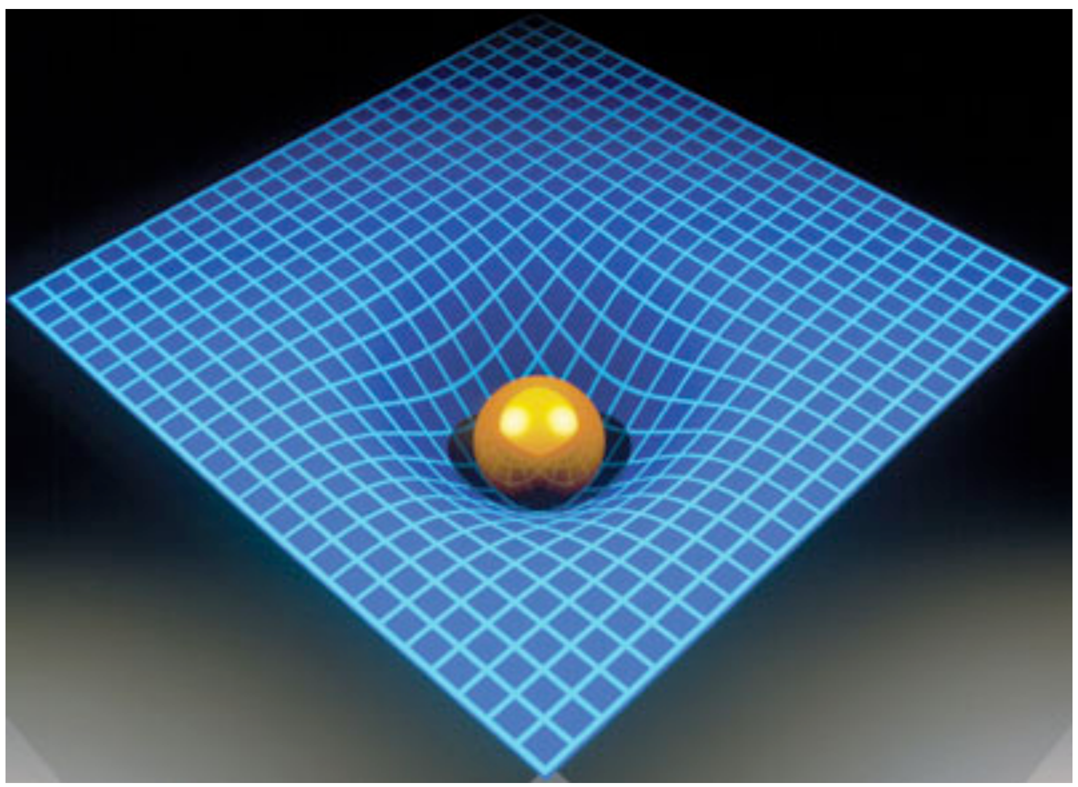




Cassini: radio delay by curved space around Sun

Einstein’s theory of gravitation (1915)

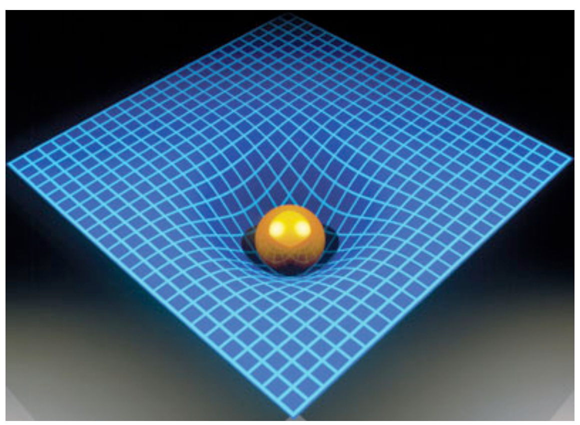
(General relativity)



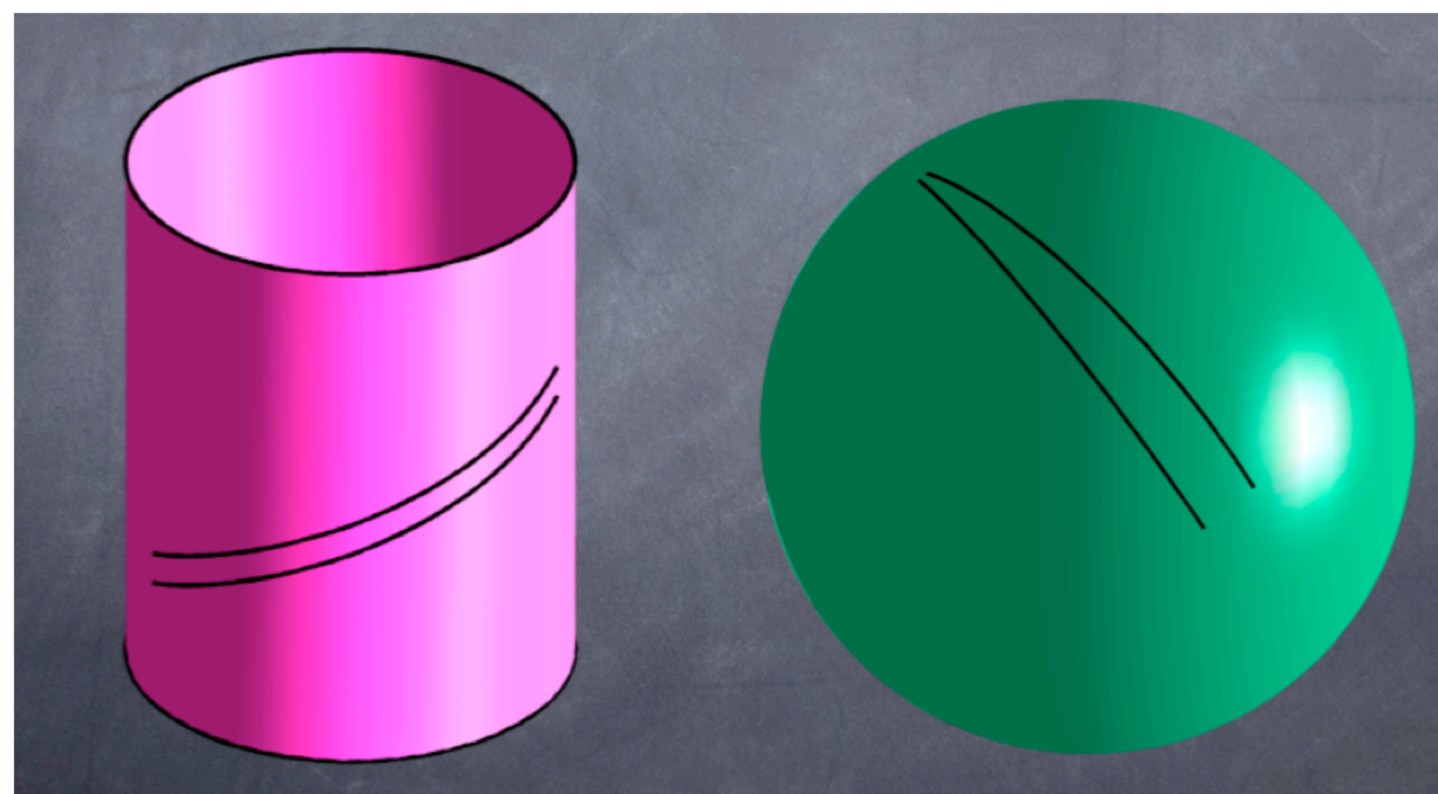
Mathematical description?

Einstein's theory of gravitation (1915)

(General relativity)



Geometry of surfaces:
intrinsic curvature



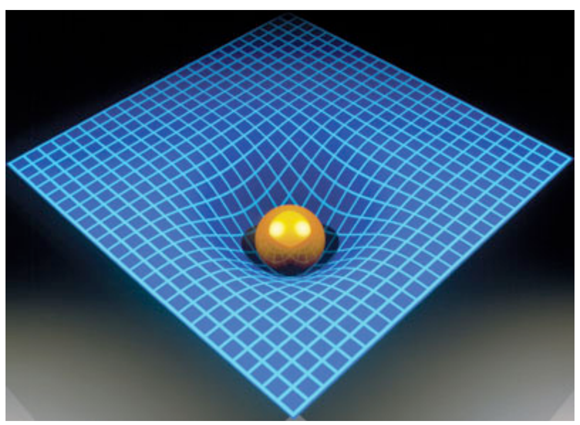
a "metric"

$$(g_x(x,y), g_y(x,y))$$

$$ds^2 = g_x dx^2 + g_y dy^2$$

Einstein's theory of gravitation (1915)

(General relativity)



Geometry of surfaces:
intrinsic curvature

a "metric"

$$(g_x(x,y), g_y(x,y))$$

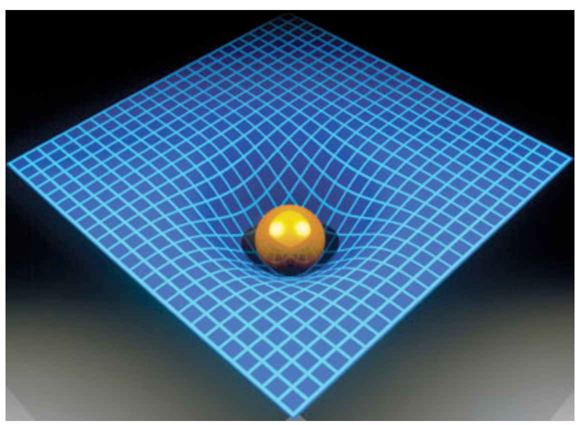
$$ds^2 = g_x dx^2 + g_y dy^2$$

curvature:
derivatives of metric

$$\frac{\partial^2 g_x(x,y)}{\partial x^2} + \ldots$$

Einstein's theory of gravitation (1915)

(General relativity)



Geometry of *four*-dimensional spacetime:
uses Riemannian geometry (1868)


Bernhard Riemann

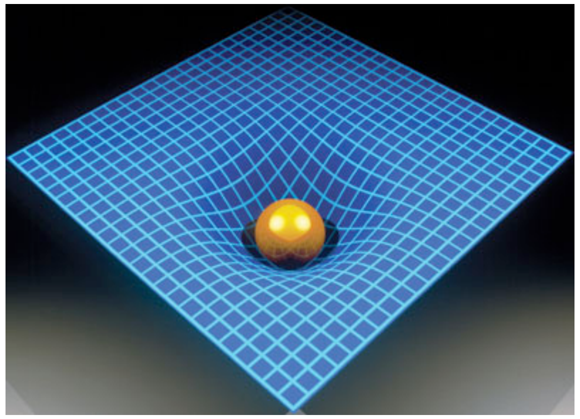
$$(g_x(x,y,z,t), g_y(x,y,z,t),$$
$$g_z(x,y,z,t), g_t(x,y,z,t))$$

$\mathbf{g}$

Einstein's equations

$$\mathbf{G}(\mathbf{g}) = 8\pi G_N \mathbf{T}$$

curvature matter



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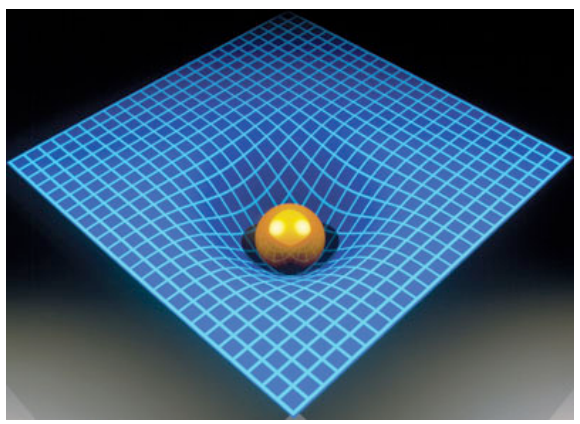


useful alternative: Hilbert action (Einstein's equation as Euler-Lagrange equations)

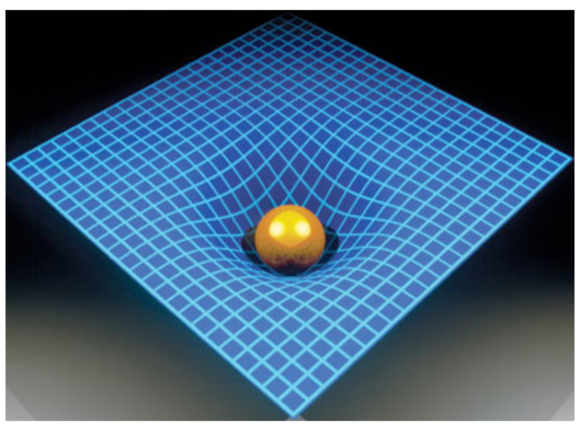
$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} R \qquad \left(M_P^2 = \frac{1}{8\pi G_N} \right)$$

curvature

Einstein's equations and Newton

$$\mathbf{G}(\mathbf{g}) = 8\pi G_N \mathbf{T}$$

curvature matter



$$g_t = -(1 + 2\phi)$$

$$\nabla^2 \phi = 4\pi G_N \cdot \rho$$

Einstein's theory reduces to Newton's for small velocities and small masses

Tests of Einstein's theory

Will '05
...
Trodden '10

- solar system, e.g. bending of light
- pulsars, e.g. loss of energy
- nucleosynthesis, e.g. helium production
- expansion of the universe (not really)

Tests of Einstein's theory

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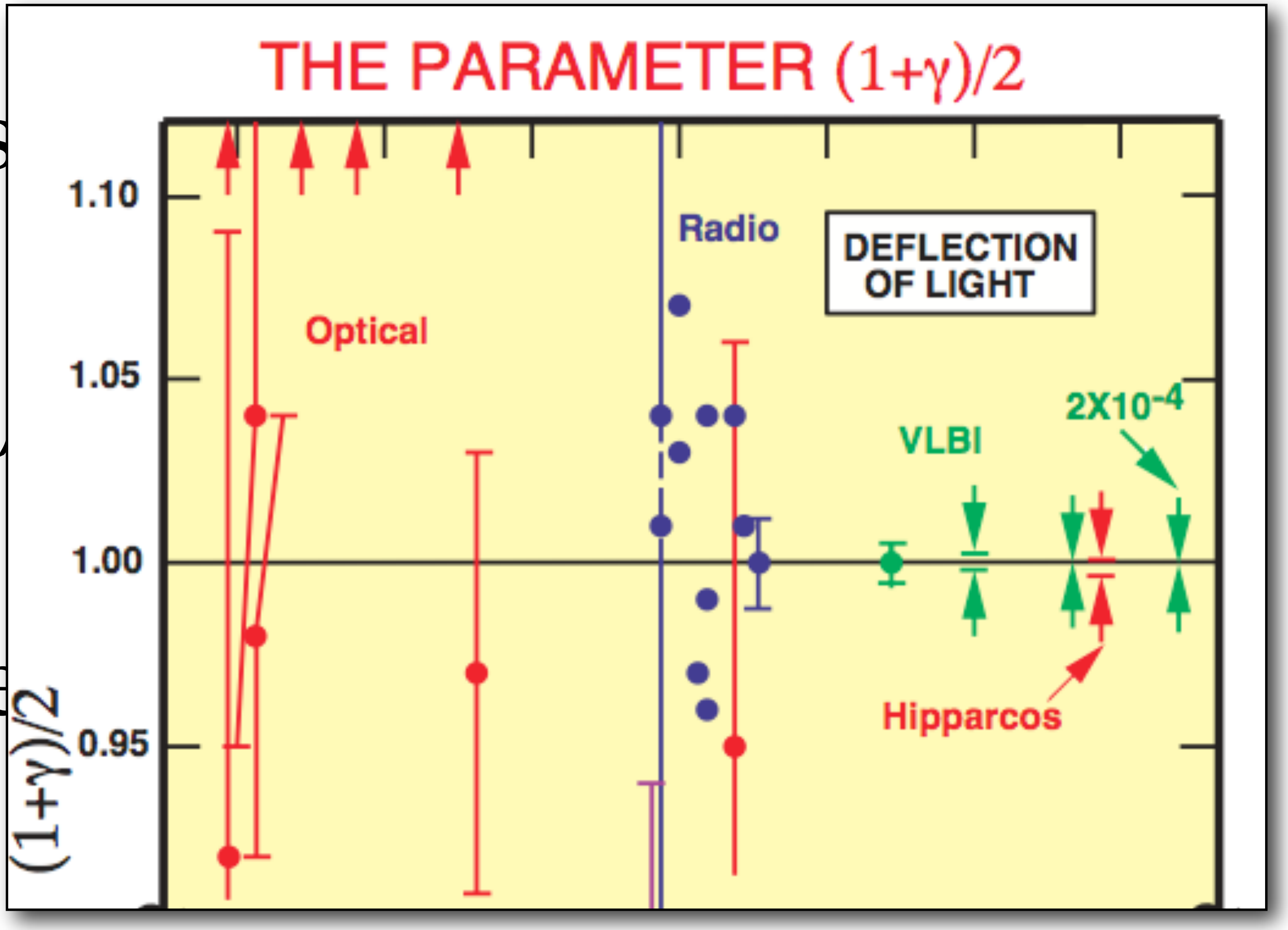

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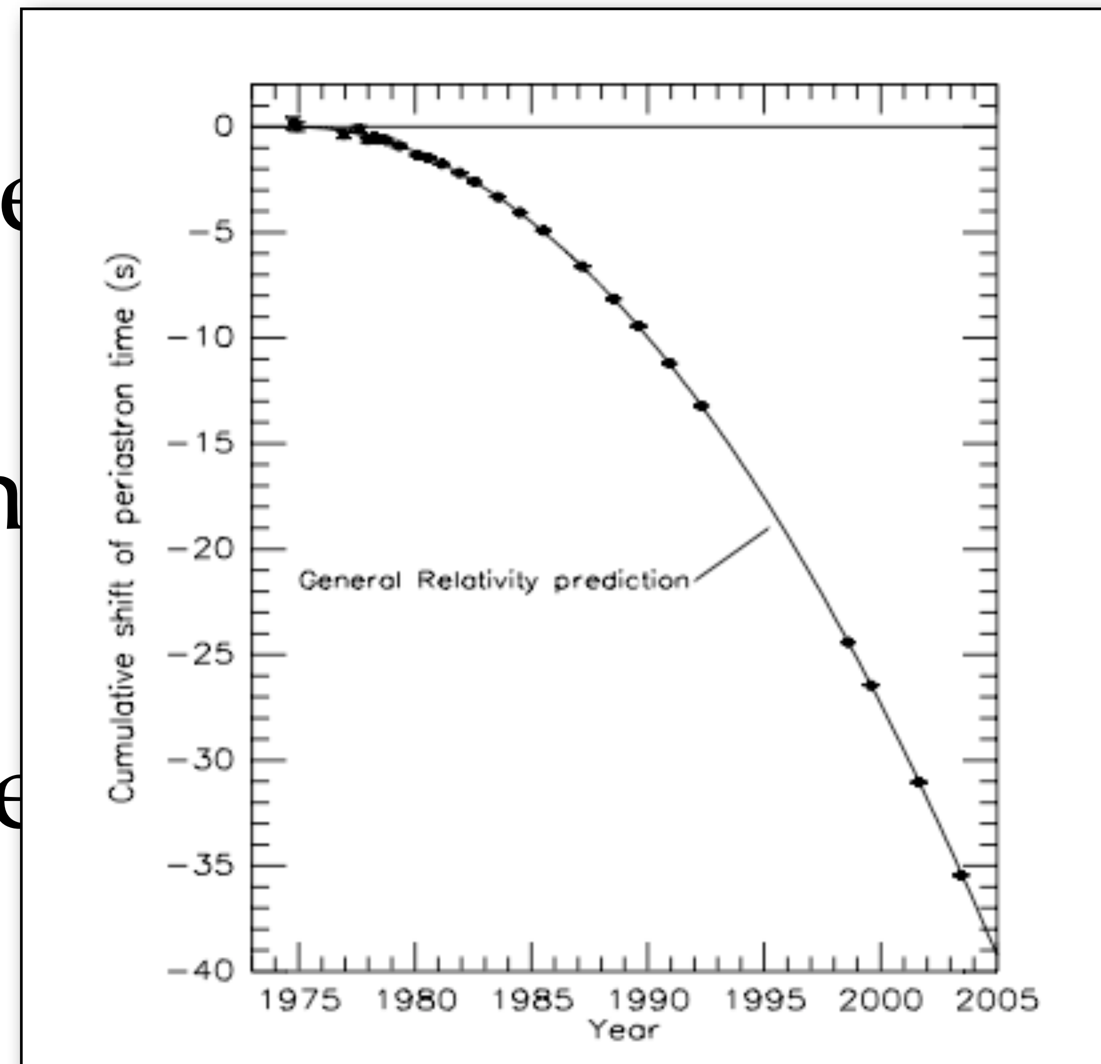

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Tests of Einstein's theory

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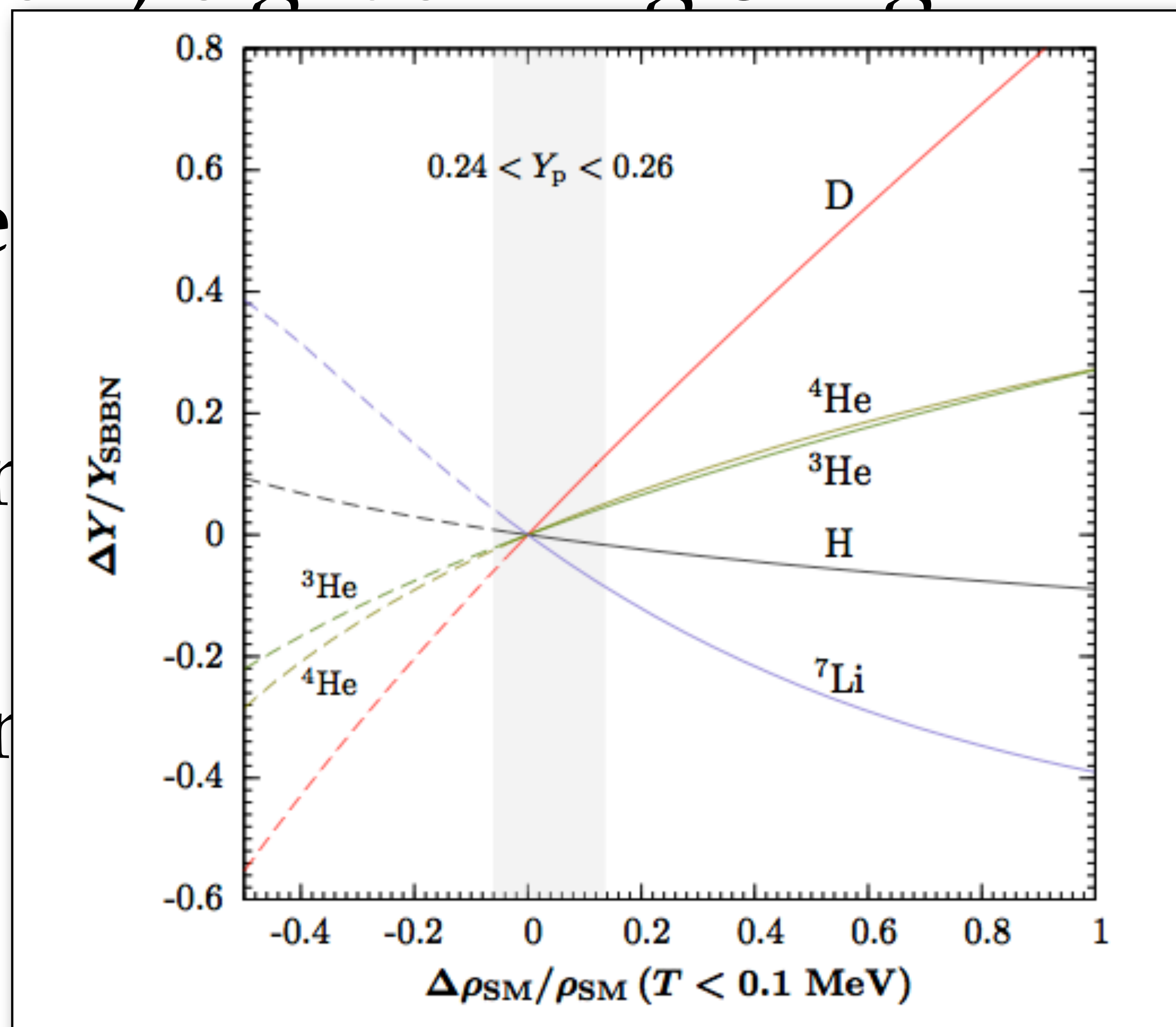
Trodden '10

- solar system, e.g. bending of light
- pulsars, e.g. loss of energy
- nucleosynthesis, e.g. helium production, 24%
- expansion of the universe (not really)

Tests of Einstein's theory

Will '05
...
Trodden '10

- solar system, e.g. bending of light
- pulsars, e
- nucleosyn ... on, 24%
- expansion



Solar system, modern

Trodden '10

- Laser lunar ranging
- GPS satellites (mostly special relativity)

Solar system, modern

Trodden '10

- Laser lunar ranging: 38 mm / year,
  G_N: 10^{-11} since 1969

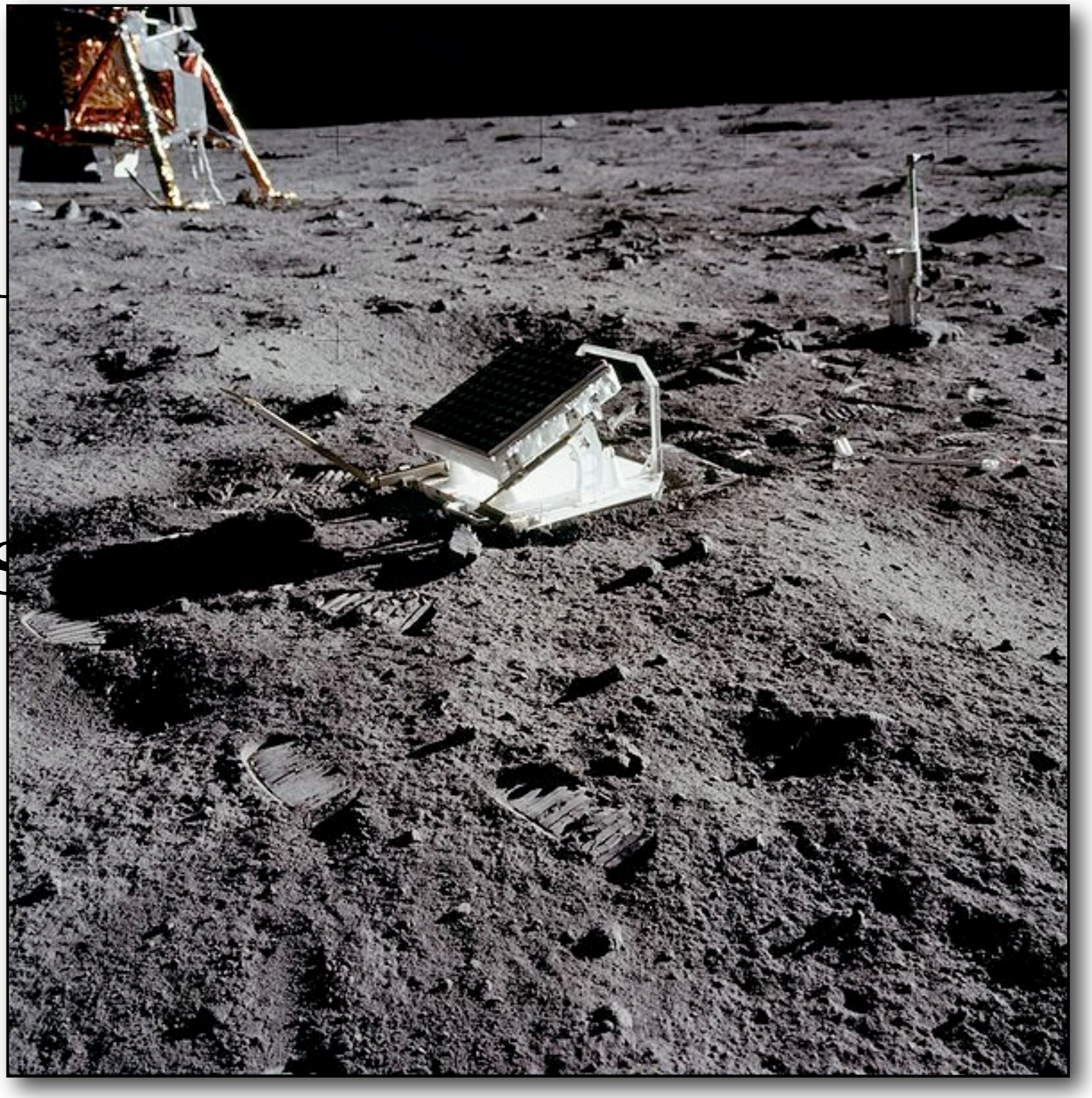
- GPS satellites (mostly special relativity)

Solar system, modern

Trodden '10

- Laser lunar rangin

- GPS satellites (mos



Solar system, modern

Trodden '10

- Laser lunar ranging
- GPS satellites (mostly special relativity)

Problems with Einstein's gravitation

(• solar-system anomalies, like Pioneer anomaly)

(• galaxy rotation curves: solved by dark matter)

(• structure formation: too many subhaloes
maybe just haven't seen them yet)

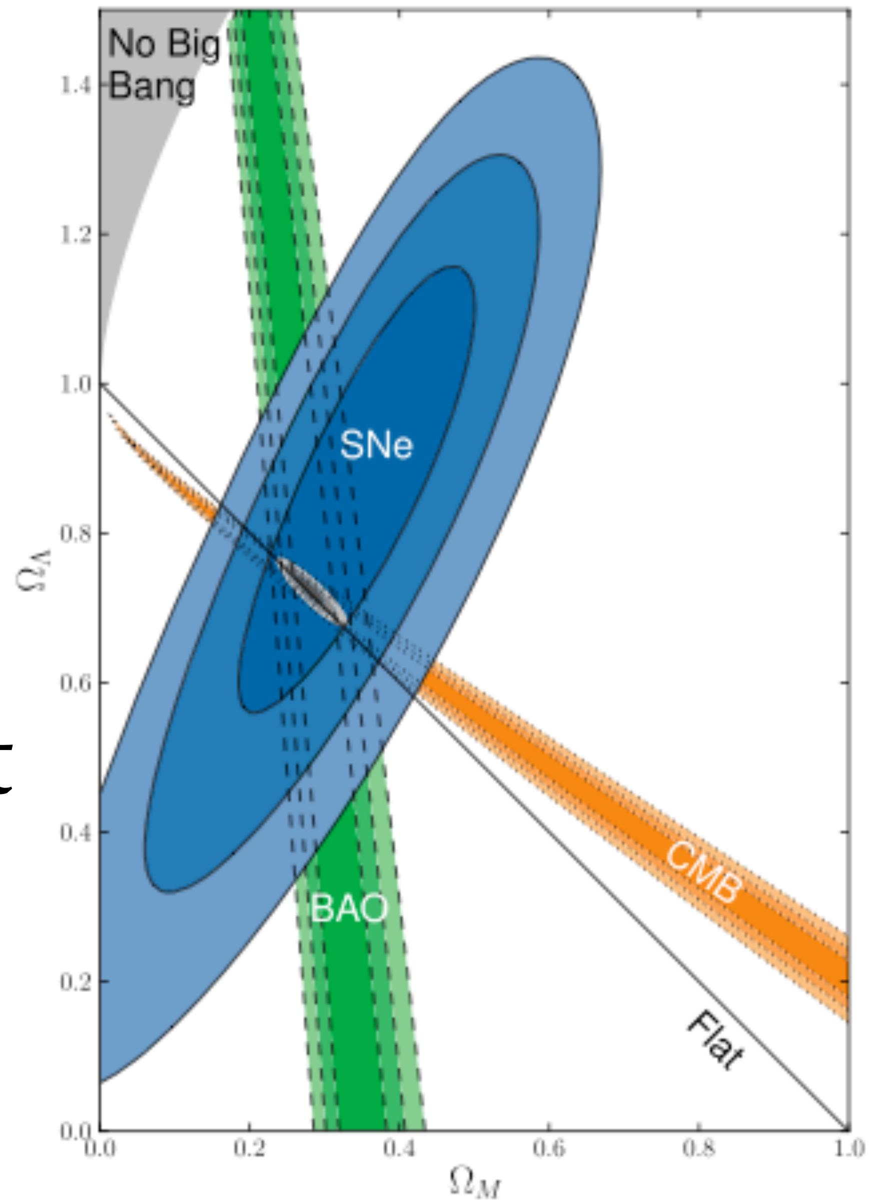
• cosmic acceleration: dark energy
(not a "problem" per se, but a mystery)

• quantum theory of gravity?

One motivation for modified gravity: "understanding cosmic acceleration"

Amanullah et al (2010)

"dark energy"
~ 70%
of universe
energy budget



cosmic acceleration seems to be here to stay

One motivation for modified gravity: "understanding cosmic acceleration"

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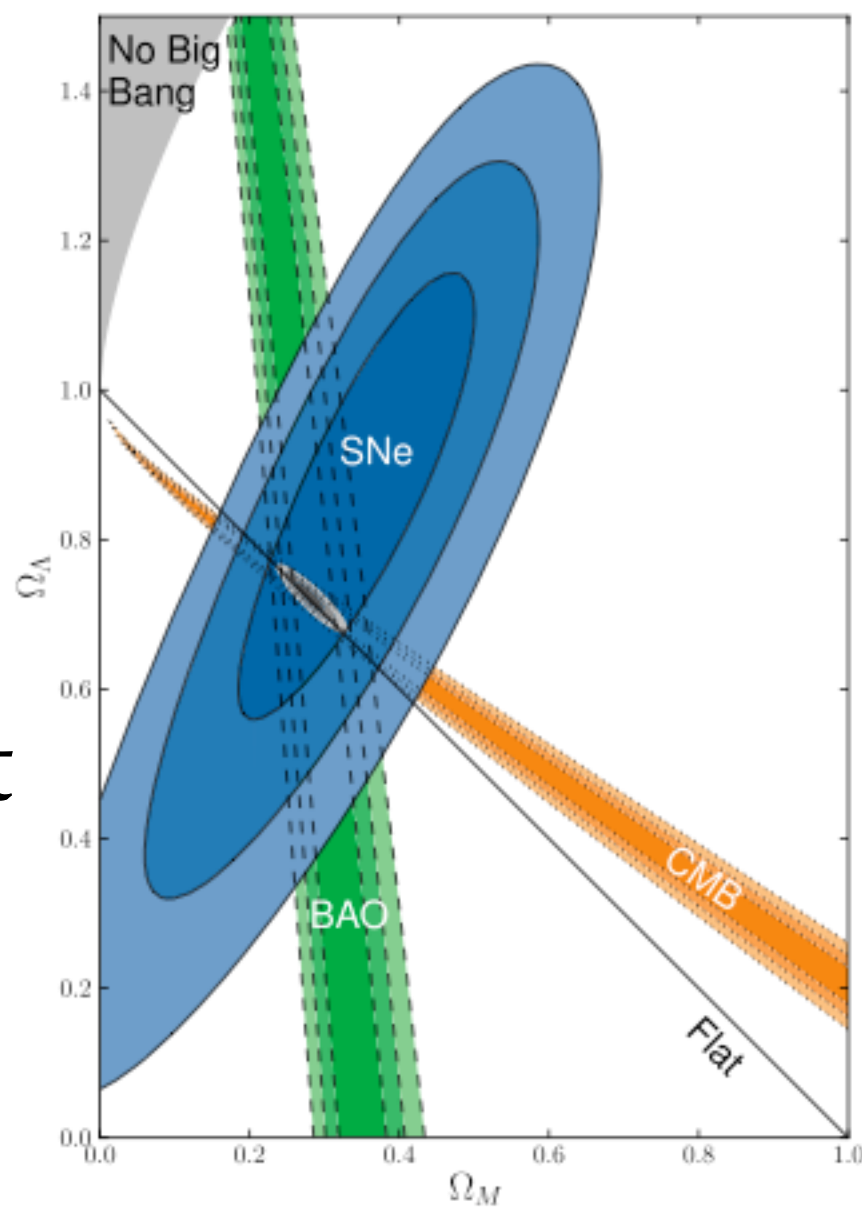
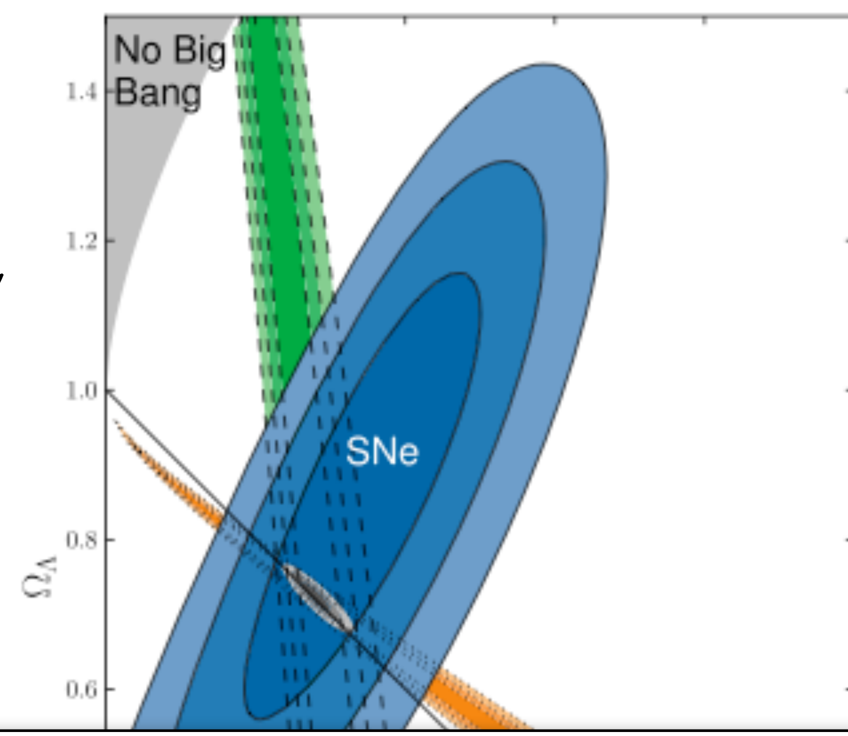


Amanullah et al (2010)

NASA's
BEYOND
EINSTEIN
PROGRAM

cosmic acceleration seems to be here to stay

One motivation for modified gravity: "understanding cosmic acceleration"



Amanullah et al (2010)

"dark energy"
~ 70%
of universe
e

NASA's
BEYOND

The story might stop right here with a happy ending —a complete physics model of the cosmic expansion— were it not for a chorus of complaints from the particle theorists. (Perlmutter, 2003)

One motivation for modified gravity: "understanding cosmic acceleration"

- Quantum theory describes the rest of the world, why should cosmology get away without? Noone really knows how, but let's try.

- Not just energy *differences* matter in Einstein's theory, but *absolute* energy. So, also the quantum energy of the vacuum?

$$E_n = \hbar\omega \left(n + \frac{1}{2} \right)$$

$$E_{\rm theory} \gtrsim 1\,\mathrm{TeV}$$

$$E_{\rm exp} \sim 10^{-3}\,\mathrm{eV}$$

One motivation for modified gravity: "understanding cosmic acceleration"

- Two obvious possible solutions:
  - modify gravity
  - modify quantum theory

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

$$E_{\rm theory} \gtrsim 1\,{\rm TeV}$$

$$E_{\rm exp} \sim 10^{-3}\,{\rm eV}$$

naive!

Modified gravity

- scalar-tensor

  Brans-Dicke (1961)

- extra dimensions

  Dvali-Gabadadze-Porrati (2000)

- massive gravity

  Fierz-Pauli (1939), ... ,

  de Rham-Gabadadze (2010)

Here, modified gravity is NOT...

- ... higher derivative (truncated)
  – typically only differs appreciably from Einstein gravity when it becomes inconsistent

- ... f(R) gravity
  – no effective field theory formulation

- ... anything else than the specific theories I discuss (there are a lot!)

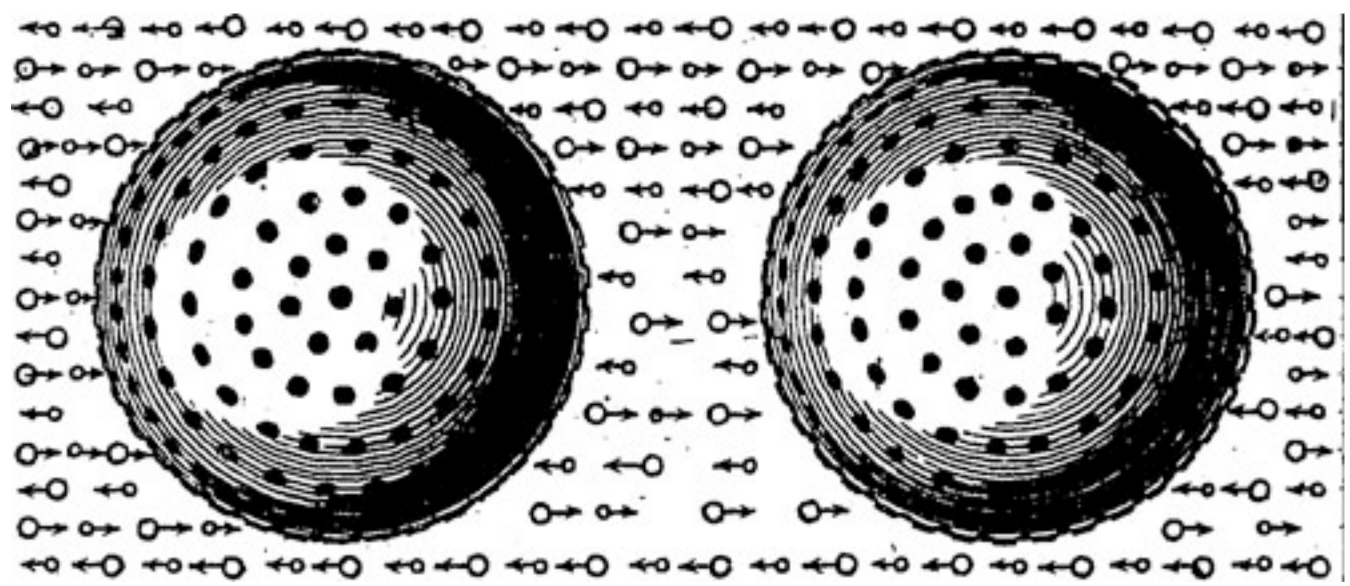
Brans-Dicke (1961)

Even though Brans-Dicke theory is "old", there were many alternatives to Einstein before Brans-Dicke

In fact, there were many alternatives to Newton before Einstein! ....

Le Sage's theory of gravitation (1748)

"ultramundane corpuscles"
"gravity particles" that create force of gravity by "pushing"



(doesn't work)

Brans-Dicke

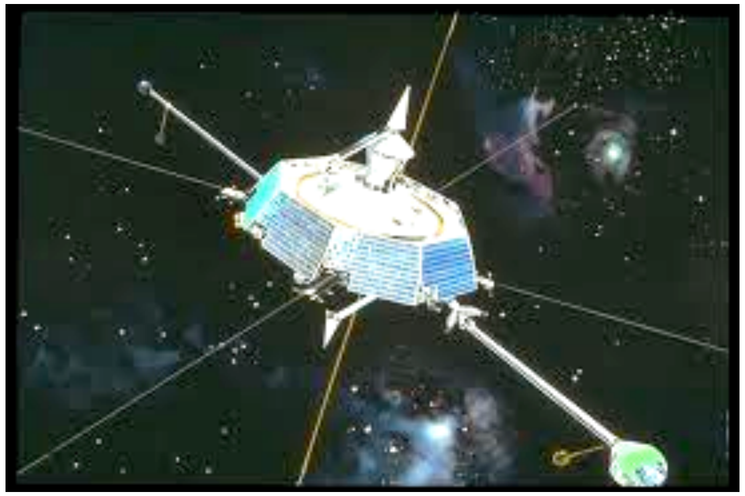
$$S_{\rm BD} = \frac{M_{\rm Pl}^2}{2} \int {\rm d}^4x \sqrt{-g} \left(\Phi R - \frac{\omega_{\rm BD}}{\Phi} (\partial\Phi)^2 \right) + \int {\rm d}^4x \sqrt{-g} \mathcal{L}_{\rm matter}[g]$$

Gravitational field described not just by metric $\mathbf{g}$ but also by scalar field Φ

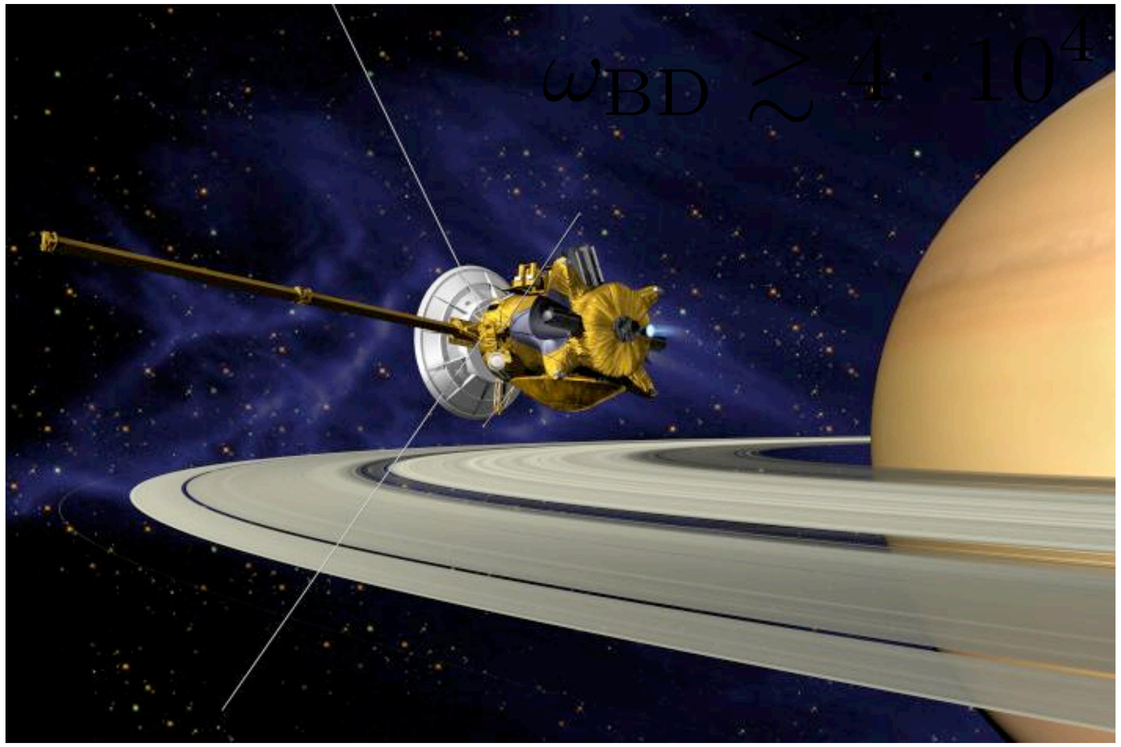
Brans-Dicke

$$S_{\rm BD} = \frac{M_{\rm Pl}^2}{2}\int {\rm d}^4x\sqrt{-g}\left(\Phi R - \frac{\omega_{\rm BD}}{\Phi}(\partial\Phi)^2\right) + \int {\rm d}^4x\sqrt{-g}\mathcal{L}_{\rm matter}[g]$$

Effectively, this is like variation in Newton's constant G_N, that can be tested



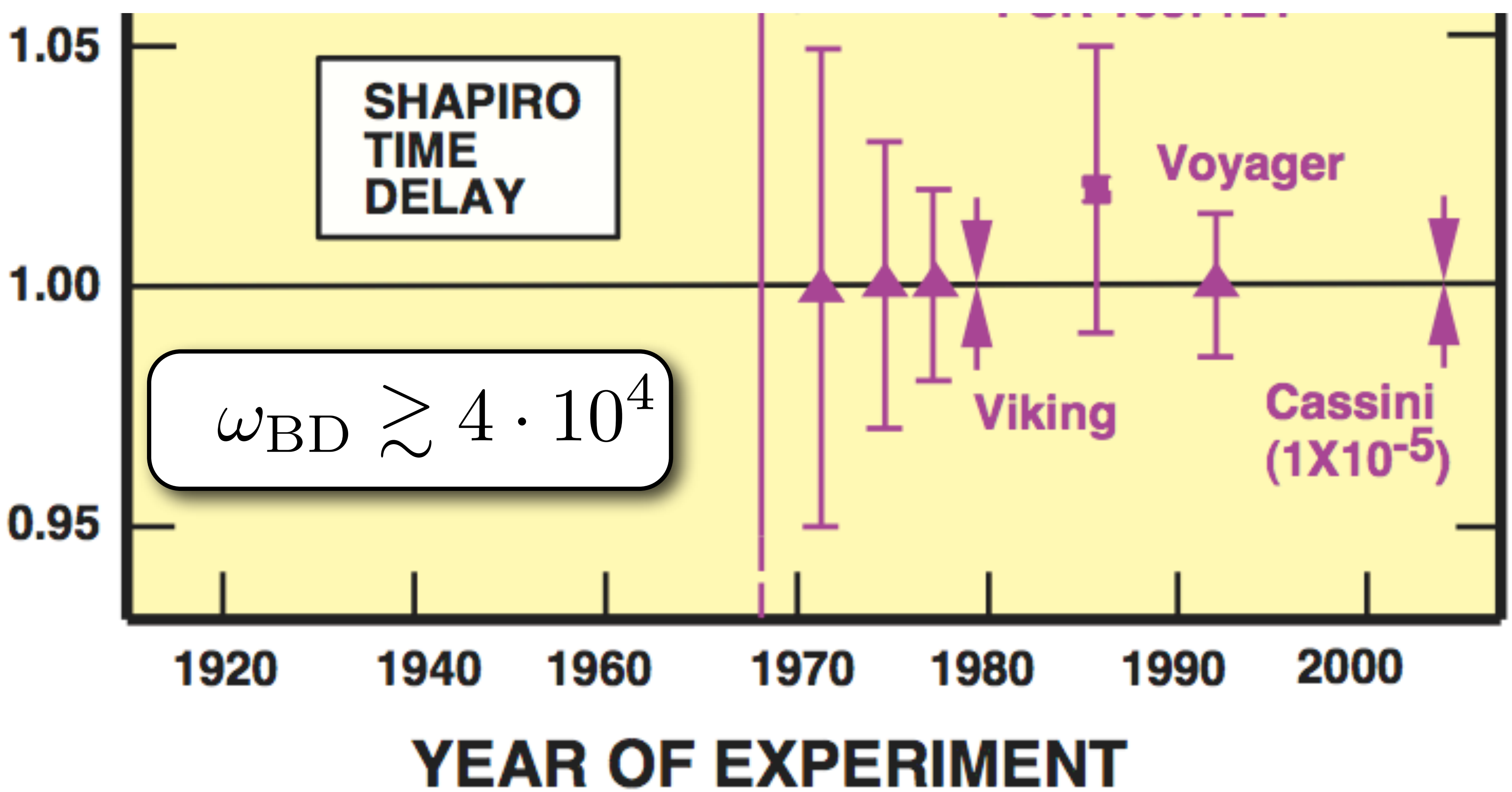
Viking



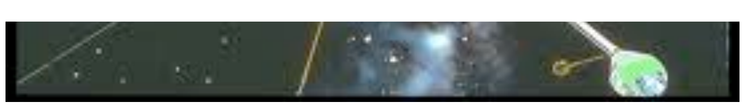
Cassini

Brans-Dicke

$$S_{\mathrm{BD}} = \frac{M_{\mathrm{Pl}}^2}{2} \int \mathrm{d}^4x \sqrt{-g} \left(\Phi R - \frac{\omega_{\mathrm{BD}}}{\Phi} (\partial \Phi)^2 \right) + \int \mathrm{d}^4x \sqrt{-g} \mathcal{L}_{\mathrm{matter}}[g]$$



$$\omega_{\mathrm{BD}} \gtrsim 4 \cdot 10^4$$



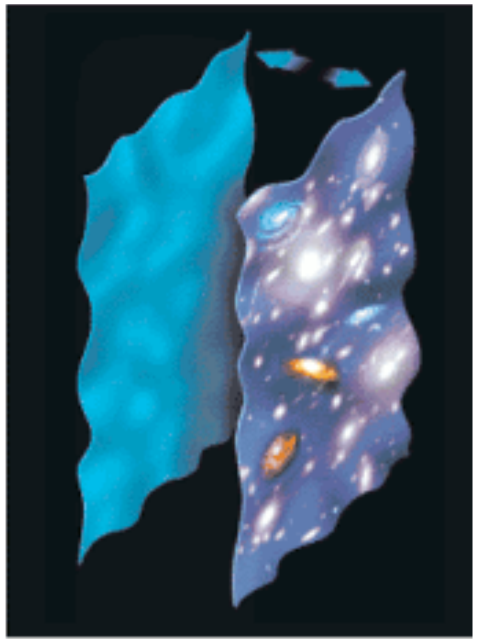
Viking

Cassini

Dvali-Gabadadze-Porrati model (DGP)

$$S_{\rm DGP} = \int_{\rm bulk} {\rm d}^5x\sqrt{-g_5}\frac{M_5^3}{2}R_5 + \int_{\rm brane} {\rm d}^4x\sqrt{-g_4}\left(\frac{M_{\rm Pl}^2}{2}R_4 + \mathcal{L}_{\rm matter}[g]\right)$$

"brane world"



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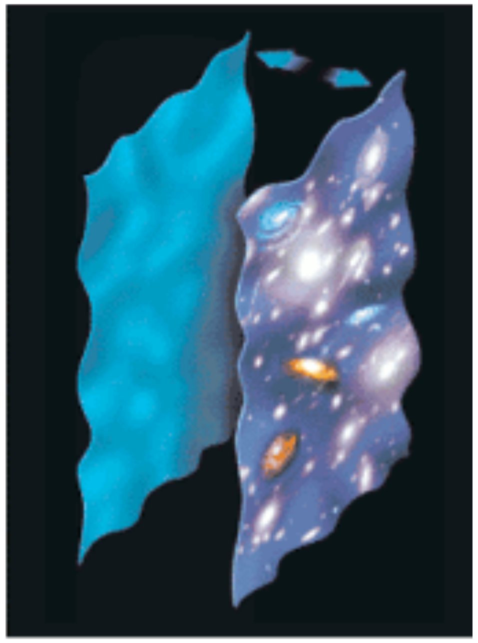

"brane bending mode" π

Dvali-Gabadadze-Porrati model (DGP)

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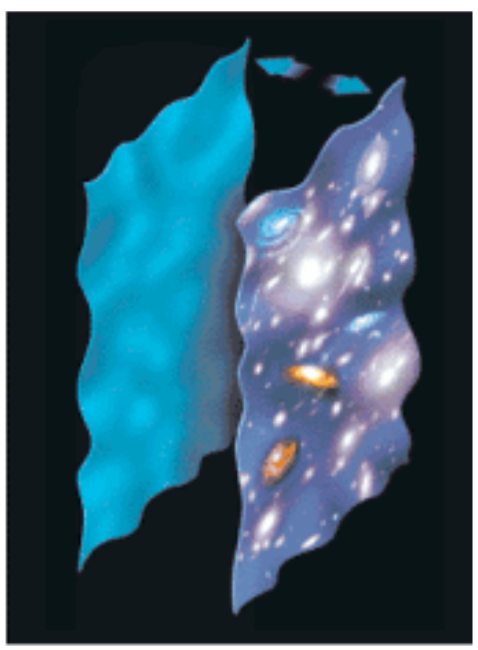
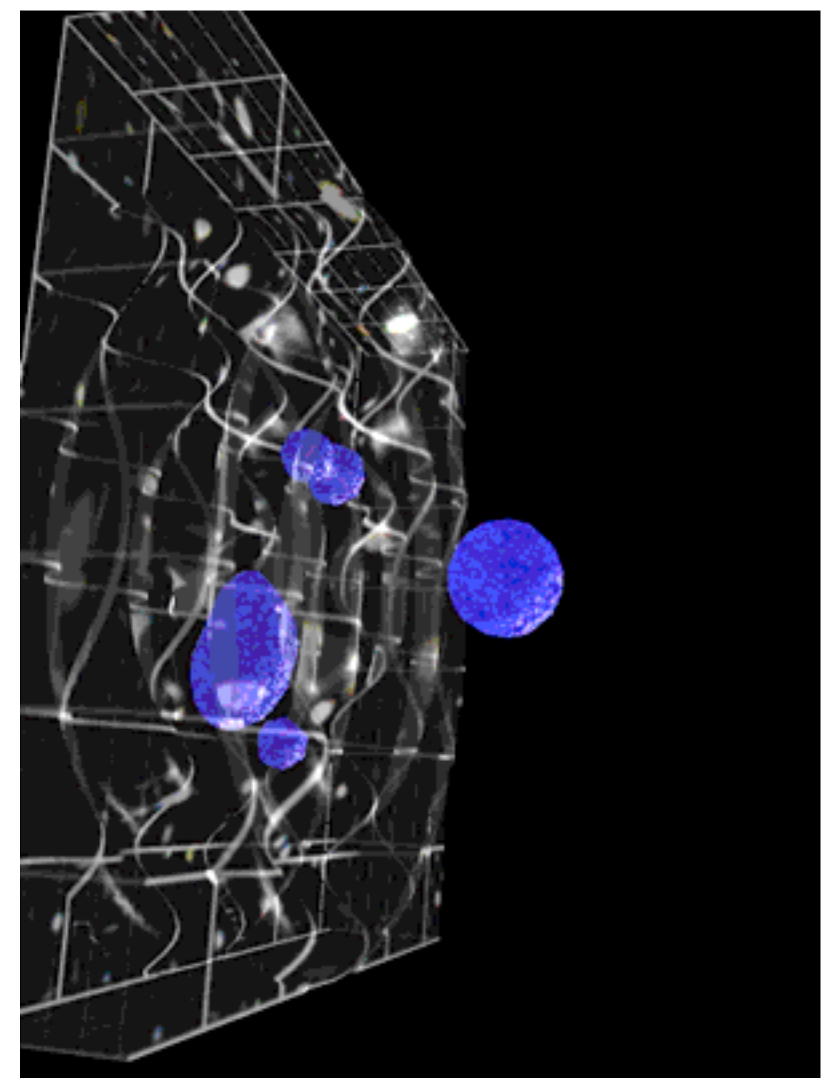
"brane world"





"brane bending mode" π

$\longrightarrow$ particle physics, too

Dvali-Gabadadze-Porrati model (DGP)

$$S_{\mathrm{DGP}} = \int_{\mathrm{bulk}} \mathrm{d}^5x \sqrt{-g_5} \frac{M_5^3}{2} R_5 + \int_{\mathrm{brane}} \mathrm{d}^4x \sqrt{-g_4} \left(\frac{M_{\mathrm{Pl}}^2}{2} R_4 + \mathcal{L}_{\mathrm{matter}}[g] \right)$$

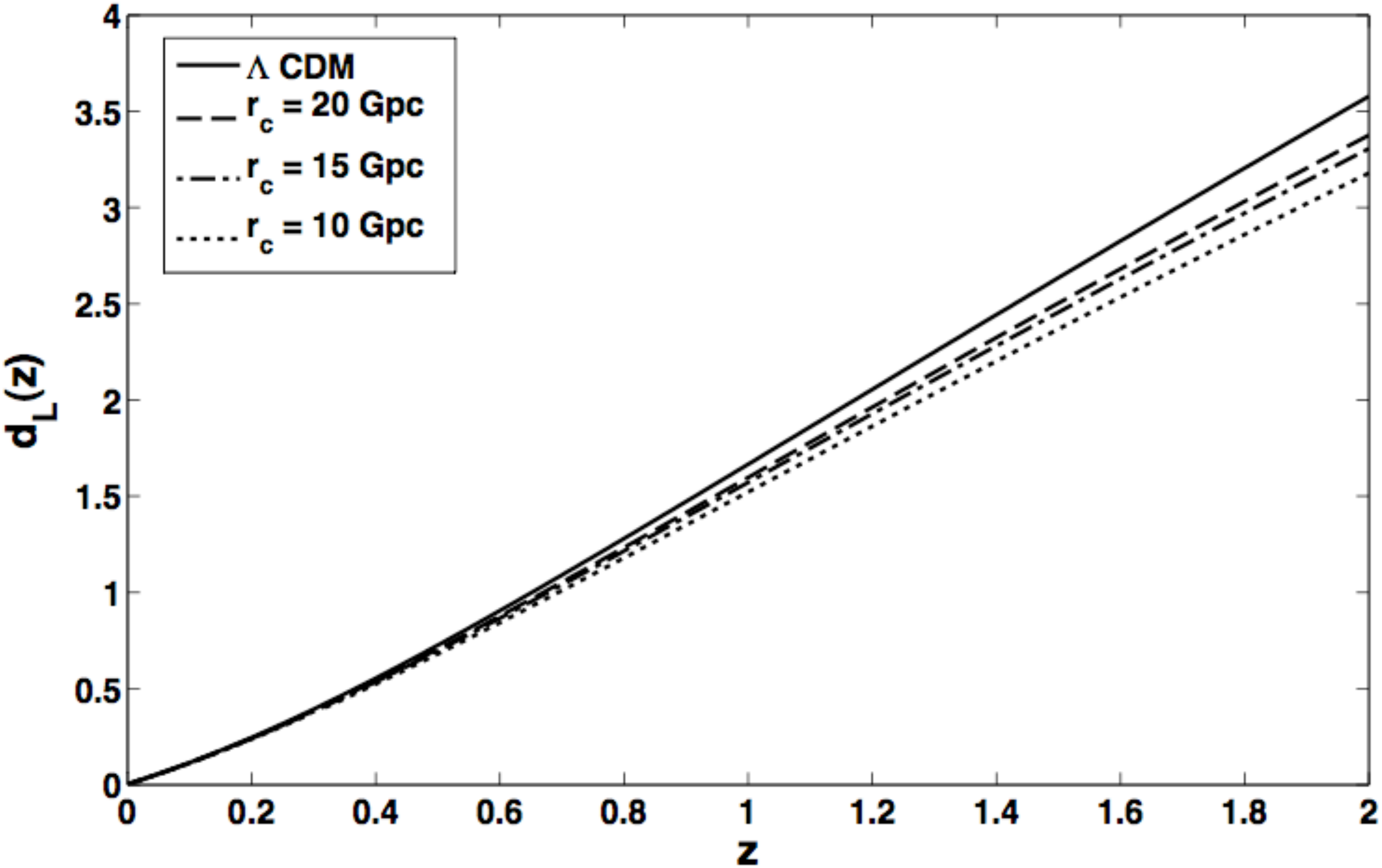
Chow, Khoury '10

Simplified DGP, effectively like Brans-Dicke,
but scalar π "frozen" in solar system,
so effect is *negligible,*
while parameters are *not too restricted*

DGP

$$S_{\rm DGP} = \int_{\rm bulk} {\rm d}^5x\sqrt{-g_5}\frac{M_5^3}{2}R_5 + \int_{\rm brane}{\rm d}^4x\sqrt{-g_4}\left(\frac{M_{\rm Pl}^2}{2}R_4 + \mathcal{L}_{\rm matter}[g]\right)$$

Chow, Khoury '10



$\sim 10\,{\rm Gyr}$

DGP

$$S_{\mathrm{DGP}} = \int_{\mathrm{bulk}} \mathrm{d}^5x\sqrt{-g_5}\,\frac{M_5^3}{2}R_5 + \int_{\mathrm{brane}} \mathrm{d}^4x\sqrt{-g_4}\left(\frac{M_{\mathrm{Pl}}^2}{2}R_4 + \mathcal{L}_{\mathrm{matter}}[g]\right)$$

Chow, Khoury '10

The DGP model has trouble with
the formation of structure (galaxies, ...)

... and it is being abandoned by most of its
creators for theoretical reasons.

Massive gravity

Fierz, Pauli (1939)
...
van Dam, Veltman (1970)
...
Vainshtein (1972)

the photon is the particle that transmits light

the *graviton* is the particle that transmits gravity

Massive gravity

Fierz, Pauli (1939)
...
van Dam, Veltman (1970)
...
Vainshtein (1972)

"massive gravity" means:
allow for a small mass for the graviton

this is a modification of gravity, irrespective of whether we detect individual gravitons

Massive gravity

Fierz, Pauli (1939)
...
van Dam, Veltman (1970)
...
Vainshtein (1972)

- a (massless) *photon* has 2 polarization states
- a "massive photon", like the Z boson, has 3 polarization states

- a massless *graviton* has 2 states
- a massive graviton has 5 states, but 2 will not concern us here , 1 left: π

Massive gravity

7.A.1 Nuclear Physics B22 (1970) 397-411. North-Holland Publishing Company

MASSIVE AND MASS-LESS YANG-MILLS AND GRAVITATIONAL FIELDS

H. van DAM * and M. VELTMAN
Institute for Theoretical Physics, University of Utrecht, Utrecht, The Netherlands

Received 8 June 1970

Ab

Thus in the massive case (but with extremely small mass) the bending of a ray of light passing near the sun is $\frac{3}{4}$ of that predicted in the mass-less case. Experiment is however too vague to decide between the two cases*.

* *Note added in proof.* Recently more precise experiments have been performed [9], agreeing closely with Einstein's theory, thereby excluding the massive theory.

Massive gravity

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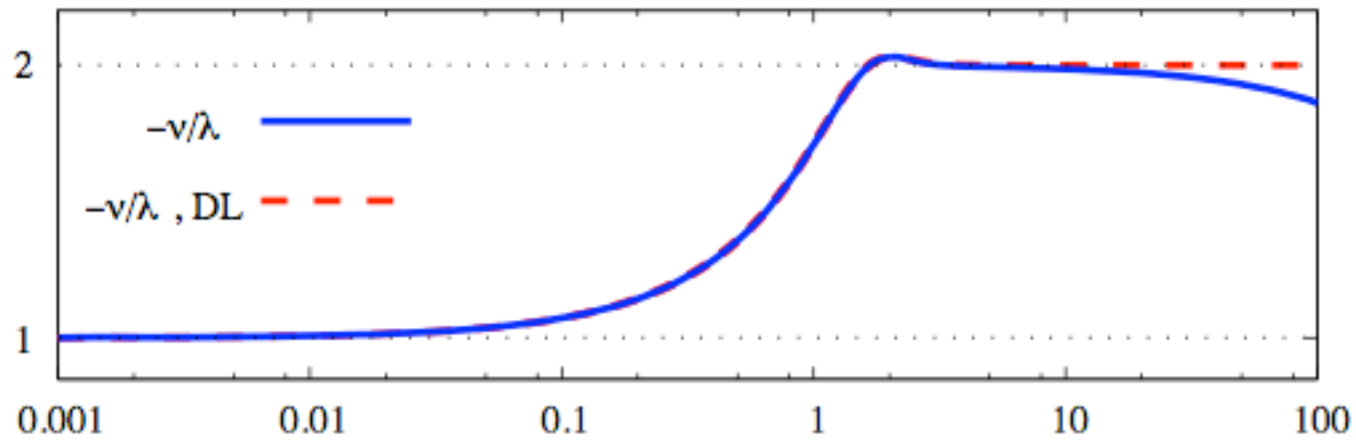
Veltman
(Nobel 1999, quantum massive Yang-Mills fields)

Massive gravity

Babichet, Deffayet, Ziour (2010)

Metric functions / gravitational potentials



R/R_V

Vainshtein radius

$$R_V = \left(R_S \lambda_g^2\right)^{1/3}$$

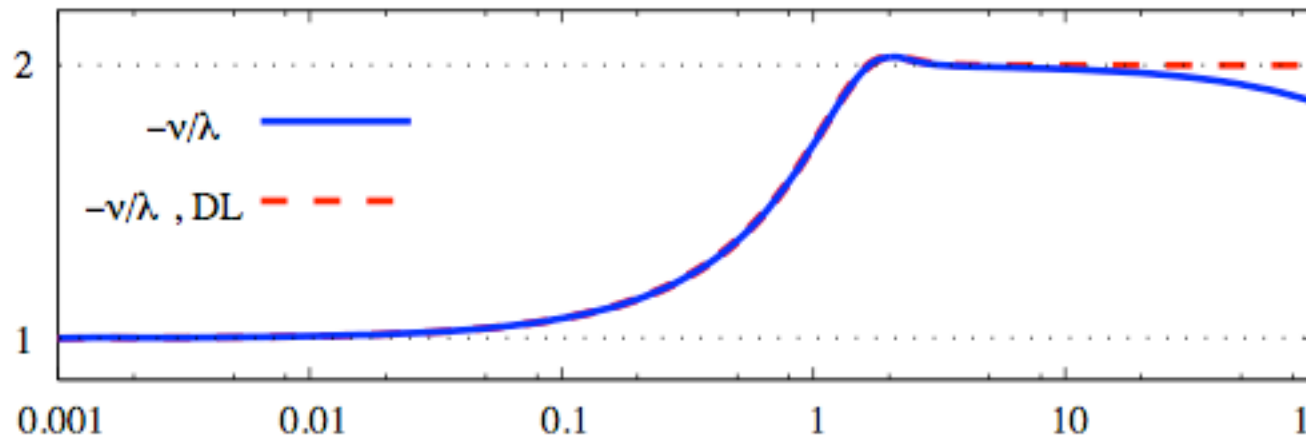
for the sun, this is about 120 parsec

(1 parsec ~ 200000 AU)

Massive gravity

Babichet, Deffayet, Ziour (2010)

Metric functions / gravitational potentials



R/R_V

Vainshtein radius

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for the sun, this is about 120 parsec

(1 parsec ~ 200000 AU)

no difference in solar system

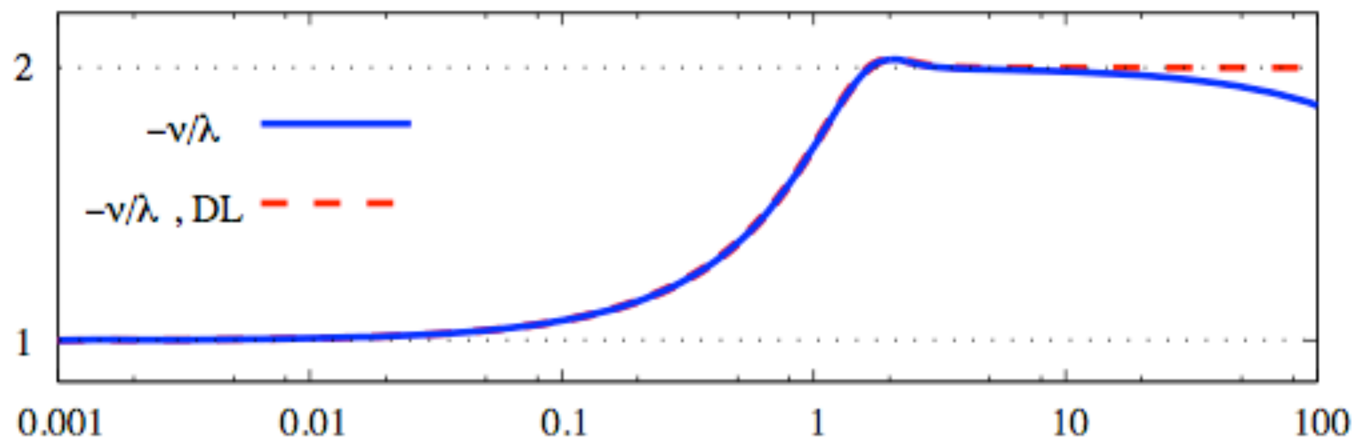
could make a difference at

- medium distance (clusters)
- very long distance (expansion history)

Massive gravity

Babichet, Deffayet, Ziour (2010)

Metric functions /
gravitational potentials



R/R_V

Vainshtein radius

$$R_V = \left(R_S \lambda_g^2\right)^{1/3}$$

perhaps also “Vainshtein time”
– theory OK with nucleosynthesis
– massive gravity kicks in sometime
when dark energy kicks in

Modified gravity, outlook

Many models, here focused on ones that:

- make pretty clear predictions
- no internal theoretical problems (?)
- are not ruled out (until now?)
- have bearing on dark energy problem
  – might be possible to quantize

String theory: the next synthesis?

- can quantize gravity
- has extra dimensions
- has branes
- might give massive gravity?

"Superstring Modification of Einstein's equations"
(Gross, Witten 1986)

String theory: the next synthesis?

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Better chance than GW:

Brane Induced Gravity, its Ghost and the Cosmological Constant Problem

S. F. Hassan[a*], Stefan Hofmann[b,c†] and Mikael von Strauss[a‡]

[a] *Department of Physics & The Oskar Klein Centre for Cosmoparticle Physics, Stockholm University, AlbaNova University Centre,*

"Superstring Modification of Einstein's equations"

(Gross, Witten 1986)

Summary

- let's be clear: no immediate crisis
  for Einstein's theory

- a handful of observations seem
  to give hints that maybe there is a problem

- conceptual problems:
  cosmic acceleration, quantum theory

- proposed improvements: BD, DGP, massive

Summary

- let's be clear: no immediate crisis
  for Einstein's theory

- a handful of observations seem
  to give hints that maybe there is a problem

- conceptual problems:
  cosmic acceleration, quantum theory

- proposed improvements: (BD), DGP?, massive

Docentföreläsning

THE END

Thanks for listening!

Stockholm University

Marcus Berg
CoPS
OKC

Oskar Klein centre